বিদ্যাসাগর বিশ্ববিদ্যালয়

# VIDYASAGAR UNIVERSITY

## Question Paper

## B.Sc. Honours Examinations 2021

**(Under CBCS Pattern)**

## Semester - V

## Subject : MATHEMATICS

**Paper : DSE 2 - T**

**Full Marks : 60**

**Time : 3 Hours**

*Candidates are required to give their answers in their own words as far as practicable.*
*The figures in the margin indicate full marks.*



**[ PROBABILITY AND STATISTICS ]**

**Group-A**

*The Symbols have their usual meaning.*

1. Answer any ***four*** questions : 12×4=48

(a) (i) Show that the function $f(x, y)$ defined by

$$f(x, y) = \begin{cases} \sin x \sin y ; & 0 < x < \frac{\pi}{2}, 0 < y < \frac{\pi}{2} \\ 0 ; & elsewhere \end{cases}$$

is a possible two-dimensional probability density function. Find the marginal density function and prove that random variables are independent.

(ii) Test for Specified mean $H_0 : (m = m_0)$ where σ is unknown, in this case the suitable test statistic is $t = \frac{\overline{X} - m_0}{\frac{s}{\sqrt{n}}} \sim t_{n-1}$, where $s^2 = \frac{n}{n-1} S^2$ where $S^2$ is the sample variance.

(iii) If $A$ and $B$ are any two events, then prove that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$. 5+5+2

(b) (i) Prove that the modulus value of the arithmetic mean of the regression coefficients is not less than the modulus value of the correlation coefficient.

(ii) State and prove Chebyshev's inequality in convergence in probability.

(iii) A point P is taken at random on a line segment AB of length $2a$. Find the probability that the area of the rectangle $AP.PB$ will exceed $\frac{1}{2}a^2$. 5+5+2

(c) (i) A sample size 10 is drawn from each of two normal populations having the same variance, which is unknown. If the mean and variance of the sample from the first population are 7 and 26 respectively and those of the sample from second population are 4 and 10, Test at 5% level of significance that the two populations have the same mean. Given that $P(t > 2.10) = 0.025$.

(ii) Two numbers are independently chosen at random between 0 and 1. Show that the probability that their product is less than a constant $k$, $0 < k < 1$ is $k(1 - \log k)$.

(iii) Prove that for distribution function $F(x), F(-\infty) = 0$ and $F(\infty) = 1$.

5+5+2

(d) (i) Two persons A and B agree to meet at a given place between 12 noon and 1 pm with understanding that the first to arrive shall wait for 20 minutes for the other and then leave. Find the probability of meeting between A and B, if their arrival times are independent and occur at random within the agreed time interval.

(ii) Find 95% confidence interval for the mean of a normal distribution with $\sigma = 3$, given that the sample 2.3, –0.2, –0.4, –0.9 given $P(U > 1.960) = 0.025$, where $U \sim N(0,1)$ variate.

(iii) If $X$ and $Y$ are independent, then show that $E(XY) = E(X)\, E(Y)$. 5+5+2

(e) (i) Prove that the distribution function $F(x)$ has jump distribution on the left at $x = a$, provided $P(X = a) > 0$.

(ii) Find the value of constant $k$, such that $f(x) = \begin{cases} kx(1-x), & 0 < x < 1 \\ 0, & otherwise \end{cases}$ is a probability density function. Construct the distribution function and compute $P\left(X > \frac{1}{2}\right)$.

(iii) Find the mode of the Gamma distribution. 4+4+4

(f) (i) Given that the variate X is normal (0, 1). Find the variance of $e^x$.

(ii) Show that $-1 \leq \rho(X, Y) \leq 1$.

(iii) Find out the regression line of $y$ on $x$. 4+4+4

(g) (i) $F(x)$ denotes the distribution function of a continuous random variable X. Show that the expectation of X can be expressed as

$$E(X) = \int_0^\infty \{1 - F(x) - F(-x)\}\, dx$$

(ii) For any random variable X (discrete or continuous) and for any real number c, $E(|X - c|) \geq E(|X - \mu|)$ provided the expectations exist and $\mu$ is the median of X.

(iii) If $X$ and $Y$ are independent variates, X being $\chi^2$ distributed with $m$ degrees of freedom and their sum $X + Y$ is $\chi^2$ distributed with $m + n$ degrees of freedom, show that $Y$ is $\chi^2$ distributed with n degrees of freedom. 5+4+3

(h) (i) If $(x_1, x_2, \ldots\ldots, x_n)$ be any random sample of size $n$ drawn from normal ($m$, $\sigma$) population then the sampling distribution of the sample mean $\bar{x} = \frac{x_1 + x_2 + \ldots\ldots + x_n}{n}$ is normal $\left(m, \frac{\sigma}{\sqrt{n}}\right)$.

(ii) Let S and R be the standard deviation and range respectively of a set of n values of a variable x. Show that $\frac{R^2}{2n} \leq S \leq \frac{R^2}{4}$.

(iii) If $y = 2x$ and $x = \frac{y}{8}$ are the two regression lines of a sample $(x_i, y_i), i = 1, 2, \ldots, n$ from a bivariate population of $(X, Y)$, find the correlation co-efficient of the sample. 5+4+3

## Group-B

2. Answer any ***six*** questions : 2×6=12

(i) Define moment generating function and show that $\psi(t) = \sum_{k=0}^{\infty} \frac{\alpha_k}{k!}, t^k$.

(ii) If X is $\gamma(l)$ variate, then compute $E(\sqrt{X})$.

(iii) Prove that $\sigma(aX + b) = |\alpha|\,\sigma(X)$.

(iv) Define linear regression in a bivariate distribution.

(v) Prove that the correlation coefficient is the geometrical mean of the regression coefficients.

(vi) 100 liters of water are supposed to be polluted with $10^6$ bacteria. Find the probability that a sample of 1cc of the water is free from bacteria.

(vii) Show that the first absolute moment about the mean for the normal $(m, \sigma)$ distribution is $\sqrt{\frac{2}{\pi}}\sigma$.

(viii) Show that $\mu_{11} = \alpha_{11} - m_x m_y$.

(ix) Deine Scatter diagram to represent bivariate data.

(x) If $X$ is $N(0, 1)$ variate, then find the distribution of $e^x$.

(xi) Determine the value of the constant K which makes f(x, y) = Kxy, 0 < x < 1; 0 < y < x, a joint probability density function.

(xii) Prove that $|E\{g(X)\}| \le E\{|g(X)|\}$.

*OR*

## [ BOOLEAN ALGEBRA & AUTOMATA THEORY ]

### Group-A

*The Symbols have their usual meaning.*

1. Answer any ***four*** questions : 12×4=48

(a) (i) Give an example of an infinite lattice with finite length. 2

(ii) Show that the number of elements in a finite Boolean algebra is of power of 2. 3

(iii) Prove that there does not exist a Boolean algebra containing only three elements. 4

(iv) Prove that in a bounded distributive lattice an element can have at most one complement. 3

(b) (i) Define parse tree with an example. 2

(ii) In a finite Boolean algebra, every non zero element can be uniquely expressed as the sum of all atoms. 3

(iii) Show that any non trivial property of the recursively enumerable language is undecidable. 4

(iv) Prove that the partially ordered set $(P(X), \subseteq)$ is a lattice. 3

(c) (i) If f is a function of three Boolean variables x, y, z defined by $f(x, y, z) = xy + y'$. Express f in disjunctive normal form. 5

(ii) Define turning machine and explain programming teachniques for turning machine. 4

(iii) Prove that a lattice is modular if it satisfies

$$p \cup (q \cap (p \cup r)) = p \cup (r \cap (p \cup q)).$$ 3

(d) (i) Define modular lattice with an example. 2

(ii) In a Boolean algebra B, prove that for a, b, c in B, $(a+b+c)\cdot(a'+b+c)\cdot(a+b'+c)\cdot(a+b+c') = (b+c)\cdot(c+a)\cdot(a+b)$ 3

(iii) State and proof pumping lemma. 4

(iv) Show that the union of two recursive languages is recursive. 3

(e) (i) A committee of three persons A, B, C decides proposals by a majority of votes. A has a voting weight 3, B has a voting weight 2, and C has a voting weight 1. Design a simple circuit so that the light will glow when a majority of votes is cast in favour of the proposals. 5

(ii) Write a short note on pushdown automata. 2

(iii) Design a turning machine to accept a palindrome. 3

(iv) Define context free language. 2

(f) (i) Prove that context free languages are closed under union, concatenation and reversal. 5

(ii) Using Pumping lemma, show that language $L = \{anbncn : n \geq 1\}$ is not a context free language. 4

(iii) When is a Boolean algebra said to atomic? Give an example of it. 3

(g) (i) Give an example of regular language. 1

(ii) If L and M are regular languages then prove that L-M is also regular language.4

(iii) Show that the union of two recursive enumerable languages is also recursively enumerable. 3

(iv) Diferentiate PDA and NPDA. 4

(h) (i) Design a push down automata (PDA) to accept the set of all strings of 0's and 1's such that no prefix has more 1's than 0's. 5

(ii) Prove that in a Boolean algebra, $a + a = a$ and $a \cdot a = a$ . 3

(iii) Show that a lattice is distributive iff following identity holds $(p \cap q) \cup (q \cap r) \cup (r \cap p) = (p \cup q) \cap (q \cup r) \cap (r \cup p)$ . 4

## Group-B

2. Answer any ***six*** questions : 2×6=12

(i) What are the basic digital logic gates?

(ii) State the duality principle of Boolean algebra.

(iii) What are the limitations of Karnaugh map?

(iv) Write a short note on Halting problem.

(v) Define atom and give an example of it.

(vi) Write a note on variants of a turning machine.

(vii) What is $\in$-closure of a stage $q_0$?

(viii) Differentiate NFA and DA.

(ix) Express $E(x,y,z) = (x'+y)'+x'y$ in its complete sum-of-products form.

(x) What are the components of finite automation model?

———

*OR*

## [ PORTOLIO OPTIMIZATION ]

### Group-A

Answer any ***four*** questions : 12×4=48

1. (a) Explain how a market index is price weighted. In such a case, would you expect a $ 100 stock to be more important than a $ 25 stock? Give an example. 6

   (b) Explain how a price-weighted index and a value-weighted index adjust for stock splits. 6

2. (a) Give an example of a liquid investment and an illiquid investment. Discuss why you consider each of them to be liquid or illiquid. 4

   (b) Briely discuss the fvie fundamental factors that influence the risk premium o an investment. 8

3. (a) On August 15, you purchased 100 shares of stock in the Cara Coton Company at $ 65 a share and a year later you sold it for $61 a share. During thye year, you received dividends of $3 a share. Compute your HPR and HPY on your investment in Cara Cotton. 8

   (b) "Young people with little wealth should not invest money in risky assets such as the stock market, because they can't afford to lose what little money they have." Do you agree or disagree with this statement? Why? 4

4. (a) Your rate of return expectations for the common stock of Gray Cloud Company during the next year are : 8

| Possible rate or Return | Probability |
|---|---|
| –0.10 | 0.25 |
| 0.00 | 0.15 |
| 0.10 | 0.35 |
| 0.25 | 0.25 |

   (i) Compute the expected return $E(R_i)$ on this investment, the variance of this return ($\sigma^2$), and its standard deviation ($\sigma$).

   (ii) Under what conditions can the standard deviation are used to measure the relative risk of two investments?

(iii) Under what conditions must the coeficient of variation be used to measure the relative risk of two investments?

(b) Why is a policy statement important to the planning process? 4

5. (a) Write down the meaning and importance of NAV and how it is computed? 6

(b) What are the advantages of investing in a mutual fund? 6

6. For the Markowitz mean-variance portfolio, solve the quadratic programming problem. 12

Minimize $\frac{1}{2} w^T \sum w - \lambda m^T w$

Subject to $e^T w = 1$

Where $w = (w_1, w_2, ......, w_n)^T$, $m = (m, m_2, ....., m_n)^T$

$\mu_i = E(r_i), z = (r_1, r_2, ......, r_n)^T$, $cov(z) = \sum$

7. (a) There is a portolio having three securities A, B and C. The beta of the individual securities is 2.4, 2.1, 1.8 respectively. If the ratio of investment in these three serucities is 1:2:3, Calculate the beta of the portfolio. 6

(b) Mention the difference between capital market and money market. 6

8. (a) Ram has a portfolio with a beta of 0.82. What will be the new portfolio beta if you keep 82% of his money in the old portolio and 13% in a stock with a beta of 1.93. 8

(b) Write a short note on Portolio optimization. 4

## Group-B

Answer any *six* questions : 2×6=12

9. Explain entry load and exit load.

10. What are the diferences between portolio risk and security risk?

11. Determine the relationship between risk and return.

12. What is mutual fund?

13. What are the advantages of investing in the common stock rather than the corporate bonds of a company?

14. What is the diference between correlation and covariance between securities?

15. What is the difference between a primary and secondary capital market, and how do these two markets support each other?

16. What is holding period rate of return?

17. Explain minimum variance portfolio.

18. Discuss about Treynor Measure for portfolio performance evalation