বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

B.Sc. Honours Examination 2021

(CBCS)

1st Semester

MATHEMATICS

PAPER—GE1T

CALCULUS GEOMETRY AND DIFFERENTIAL EQUATION

Full Marks : 60

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *four* questions. 4×12

1. (a) Find the asymptotes of

$$y^3 - x^2y - 2xy^2 + 2x^3 - 7xy + 3y^2 + 2x^2 + 2x + 2y + 1 = 0$$

(b) Find the envelope of the family of lines $x\cos\alpha + y\sin\alpha = \sin\alpha\cos\alpha$ where α is a parameter. 5+7

2. (a) Show that $\int_0^{\pi/2} \sin^5 x \cos^6 x dx = \frac{8}{693}$.

(b) Find the entire area of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$.

(c) Find the length of the curve

$x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ between $\theta = 0$ and $\theta = \theta_1$.

4+4+4

3. (a) If $\lim_{x \to o} \frac{a\sin x - \sin 2x}{\tan^3 x}$ is finite, find the value of a, and the limit.

(b) Evaluate $\int \tan^5 x dx$.

(c) Show that the area bounded by one arc of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ and the X – axis is $3\pi a^2$ sq.units. 4+4+4

4. (a) Reduce the equation $8x^2 + 8xy + 2y^2 + 26x + 13y + 15 = 0$ to its cononical form.

(b) If r_1 and r_2 are two mutually perpendicular radius vectors of the ellipse $r^2 = \frac{b^2}{1 - e^2\cos^2\theta}$. Prove that $\frac{1}{r_1^2} + \frac{1}{r_2^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

Where a and b are semi-major and Semi-minor axes of the ellipse.

6+6

5. (a) Find the equation of the sphere containing the circle

$x^2 + y^2 + z^2 + 7x - 2z + 2 = 0$, $2x + 3y + 4z = 8$ as one of its great circle.

(b) Find the equation of the cone whose vertex is the origin and which has lx + my + nz = p, $ax^2 + by^2 + cz^2 = 1$ as its guiding curve.

6+6

6. (a) If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, then prove that

$$\left(x^2-1\right)y_{n+2}+(2n-1)xy_{n+1}+\left(n^2-m^2\right)y_n=0$$

(b) If α, β be the roots of the equation $ax^2 + bx + c = 0$ then show that

$$\lim_{x\to\alpha}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}=\frac{1}{2}a^2(\alpha-\beta)^2.$$

(c) Find the points of inflection of the curve x = a tan θ, y = a sinθ cosθ.

4+4+4

7. (a) A plane passing through a fixed point (a, b, c) cuts the axes at A, B, C. Show tht the locus of the centre of the sphere OABC is $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$.

(b) Find the equation of the cylinder whose generators are parallel to the straight line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and whose guiding curve is $x^2 + y^2 = 4$, z = 1.

(c) Reduce the following equation to its anomical form :

$2x^2 - 4xy - y^2 + 20x - 2y + 17 = 0$. 3+4+5

8. (a) If $y = \sin\left(m \sin^{-1} x\right)$ then show that

$$\left(1 - x^2\right) y_{n+2} - (2n+1) x y_{n+1} + \left(m^2 - n^2\right) y_n = 0$$

(b) Find the point intersection of the tangents at $\theta = \alpha$ and $\theta = \beta$ on the $conic \frac{l}{r} = 1 + e\cos\theta$.

Answer any *six* questions. 6×2

9. Examine if the ODE $e^y dx + (1 + xe^y)\, dy = 0$ is exact.

10. If the expression ax + by changes to $a^1x^1 + b^1\, y^1$ by a rotation of the rectangular axes about the origin, prove that $a^2 + b^2 = a'^2 + b'^2$.

11. Write down the equation of the sphere one of whose diameter has end points (2, –1, 3) and (0, 4, –5). Find its radins

12. Find $\lim_{x \to 0}\left(\frac{1}{x} - \frac{1}{\sin x}\right)$

13. Find the polar equation of the straight line joining the two prints $(1, \frac{\pi}{2})$ and $(2, \pi)$.

14. Find the perimeter of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$.

15. Show that $\frac{1}{3x^3y^3}$ is an integrating factor of

$y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0.$

16. Find the angle through which the axes are to be rotated so that the equation $17x^2 - 18xy - 7y^2 = 1$ may be reduced to the form $Ax^2 + By^2 = 1$, $A > 0$. Find also A, B.

17. Find the points on the conic $\frac{12}{r} = 1 - 4\cos\theta$ whose radius vector is 4.

18. Show that the curve $y = e^{-x^2}$ has points of inflexion at $x = \pm\frac{1}{\sqrt{2}}$.