DEPARTMENT OF MATHEMATICS
Jhargram Raj College
JHARGRAM:: 721507

Students' Enrichment Programme

<u>Problem of the Week</u>

(SEM - IV)

Problem Posting Date: **24.03.22** Due Date: **31.03.22**

Any One:

1. Show that for any irrational α $\lim_{n\to\infty} \sin n\alpha\pi$ does not exist.

2. Let α be irrational. Show that $A = \{m + n\alpha : m, n \epsilon \mathbb{Z}\}$ is dense in $\mathbb{R}$.

3. Given a real number α and $x \epsilon (0,1)$, calculate $\lim_{n\to\infty} n^{\alpha} x^{n}$.

……………………………………………………………………………………………

Brief Solution of the Problem Posted on 24.03.22

1. If $\lim_{n\to\infty} \sin n\alpha\pi$ exists, then $\lim_{n\to\infty} \sin(n+2)\alpha\pi$ exists. It will imply that $\lim_{n\to\infty} (\sin(n+2)\alpha\pi - \sin n\alpha\pi) = 0$. Hence $\lim_{n\to\infty} \cos n\alpha\pi = 0$ Similarly one can show that $\lim_{n\to\infty} \sin n\alpha\pi = 0$. But we know that $sin^2 n\alpha\pi + cos^2 n\alpha\pi = 1$ so we arrive at a contradiction.

2. We will show that in any interval (p,q) there exists at least one element of A.
   Let $\varepsilon = q - p$. We know that $\forall\ \alpha \in \mathbb{R} \setminus \mathbb{Q}\ \exists\ p_n , q_n \in \mathbb{N}$ such that $|\alpha - \frac{p_n}{q_n}| < \frac{1}{{q_n}^2}$ Since $\alpha \in \mathbb{R} \setminus \mathbb{Q}$ $\lim_{n\to\infty} q_n = \infty$ so $|q_n\alpha - p_n| < \frac{1}{q_n} < \varepsilon$ for almost all n. Set $a = |q_n\alpha - p_n|$, $ma \in (p,q), m \in \mathbb{Z}$.

3. $\alpha \in \mathbb{R},\ x \in (0,1)$. Now $\lim_{n\to\infty} \frac{(n+1)^\alpha x^{n+1}}{n^\alpha x^n} = \lim_{n\to\infty} x(1+\frac{1}{n})^\alpha = x < 1$. Hence the $\lim_{n\to\infty} n^\alpha x^n = 0$.