Department of Mathematics
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Class: Mathematics (Honours) Sem – 2

Tutorial Class: 01 **Topic: Sequence** **Date: 06.03.2019**

1. Let $a \geq 1$ be a real number and define $\{x_n\}$ by $x_1 = a$ & $x_{n+1} = 1 + \ln\{\frac{x_n({x_n}^2+3)}{3{x_n}^2+1}\}$ $\forall\, n \geq 1$. Show that $\{x_n\}$ is convergent. Find its limit.

2. A sequence $\{a_n\}$ is defined as $0 < a_1 < 1$ & $(2 - a_n)a_{n+1} = 1 \;\forall\, n \geq 1$.Show that $\{a_n\}$ converges to 1.

3. A sequence $\{x_n\}$ is defined as $x_{n+1} = x_n(2 - x_n) \;\forall\, n \geq 1$. where $0 < x_1 < 1$. Show that $\{x_n\}$ is convergent.

4. A sequence is defined as $= \sqrt{[\frac{ab^2+{x_n}^2}{a+1}]}$ where $x_1 = a(> 0)$& $b > a$. Show that $\{x_n\}$ is convergent in $\mathbb{R}$.

5. If $\{V_n\}$ is a sequence of positive numbers such that ${V_{n+1}}^2 = \frac{2V_n}{1+V_n}$. Show that $V_n \to 1 \;\; as \;\; n \to \infty$.

6. A sequence $\{x_n\}$ is defined as $x_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \ln n$. Show that $\{x_n\}$ is convergent.

7. Let $0 < \alpha < 1$. Let the sequence $\{x_n\}$ be defined by $x_{n+1} = \alpha x_n + (1-\alpha)x_{n-1.}$ Show that $\{x_n\}$ is convergent. Find the limit of this sequence in terms of $\alpha, x_0, x_{1.}$

8. Let $a \in \mathbb{R}^+$. The sequence $\{x_n\}$ in $\mathbb{R}$ is defined by the recurrence relation $x_{n+1} = a + {x_n}^2 \;\forall\, n \geq 0$ & $x_0 = 0$. Find NASC on $'a'$ in order that $\lim_{n\to\infty} x_n$ should exist.

9. If $\sqrt[n]{\alpha_n} = \left\{\frac{1}{2}\left(\sqrt[n]{a} + \sqrt[n]{b}\right)\right\} \;\forall\, n \in \mathbb{N}$. Show that $\alpha_n \to \sqrt{ab} \;\; as\; n \to \infty. a, b > 0$.

10. The sequence $\{a_n\}$ is defined recursively by $a_{n+1} = a_n + \frac{(-1)^n}{2^n}, a_1 \neq \frac{1}{3}$ and $b_n = \frac{2a_{n+1}-a_n}{a_n} \;\forall\, n$. Examine whether the sequences are convergent or not.

11. The Fibonacci numbers f_1, f_2 ... . are defined recursively by $f_1 = 1, f_2 = 2$ & $f_{n+1} = f_n + f_{n-1} \;\forall\; n \geq 2$. Show that $\lim_{n\to\infty} \frac{f_{n+1}}{f_n}$ exists and evaluate the limit. Is $\{f_n\}$ is convergent, if so find its limit.

12. Let $x \in \mathbb{R}\backslash\mathbb{Q}$ and let $\{\frac{m_\mu}{n_\mu}\}$ be a sequence converging to it. Show that $\{n_\mu\} \to \infty \; as \; \mu \to \infty$.

13. Let $\{x_n\}$ be a sequence in $\mathbb{R}$ & let $\{y_n\} = x_{n-1} + 2x_n \;\forall\, n \geq 2$. $y_1 = x_1$. Suppose that $\{y_n\}$ converges to $p \in \mathbb{R}$. Prove that $\{x_n\} \to \frac{p}{3} \;\; as\; n \to \infty$.

14. $\forall\; p, a \in \mathbb{R}^+$, Prove that $\lim_{n\to\infty} \frac{n^p}{(1+a)^n} = 0$.

15. If $x_1, x_2 > 0$ & $x_{n+2} = \sqrt{x_{n+1}x_n}$. Prove that $\{x_n\}$ is composed of two sequences of which one is increasing and the other is decreasing but both of them have the common limit $(x_1{x_2}^2)^{\frac{1}{3}}$.

16. If $\{f_n\}$ is a sequence of positive numbers such that $f_n = \frac{1}{2}(f_{n-1} + f_{n-2}) \forall\, n \geq 3$. Show that $\{f_n\} \to \frac{f_1+2f_2}{3}$ as $n \to \infty$.

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Reference:

1. First Course in Real Analysis by Subir Kumar Mukherjee.