Department of Mathematics
Jhargram Raj College
Jhargram

Class: Mathematics (Honours) Sem – 4

Tutorial Class: 01 Topic: Multivariable Calculus & Vector Integration. Date: 06.03.2019

1. Prove that $f(x,y) = \begin{cases} (ax+by)\sin\frac{x}{y} & ,y \neq 0 \\ 0 & ,y = 0. \end{cases}$ $x, a, b \in \mathbb{R}$ is continuous at the origin.
2. Let $f(x,y) = \frac{2xy^2}{x^2+y^4}$ $,(x,y) \neq (0,0)$. Show that $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.
3. Let $f(x,y) = \begin{cases} 1, if\ xy \neq 0 \\ 0, if\ xy = 0. \end{cases}$ Show that the repeated limit exists at the origin and are equal but the double limit does not exist at the origin.
4. Let $f(x,y) = \begin{cases} y\sin\frac{1}{x} + \frac{xy}{x^2+y^2} & ,x \neq 0 \\ 0 & ,x = 0. \end{cases}$ Show that one of the repeated limit exists but the other does not and the double limit does not exist at the origin.
5. Show that $f(x,y) = \begin{cases} xy\ln(x^2+y^2) & ,x^2+y^2 \neq 0 \\ 0 & ,\ x^2+y^2 = 0 \end{cases}$ is continuous at the origin.
6. Show that $f(x,y) = \begin{cases} e^{-\frac{|x-y|}{x^2-2xy+y^2}} & ,x \neq y \\ 0 & ,x = y \end{cases}$ is continuous at the origin.
7. Let $f(x,y) = \begin{cases} \frac{y^3}{x^2+y^2}, & x^2+y^2 \neq 0 \\ 0 & ,\ x^2+y^2 = 0. \end{cases}$ Show that (i) f_x & f_y are bounded near the origin. (ii) neither $f_x\, or\ f_y$ is continuous at the origin.
8. Let $f(x,y) = \begin{cases} \frac{x^6-2y^4}{x^2+y^2} & ,x^2+y^2 \neq 0 \\ 0 & ,\ x^2+y^2 = 0. \end{cases}$ Show that f is differentiable at the origin.
9. Let $f(x,y) = (|xy|)^p, (x,y) \in \mathbb{R}^2$.Show that f is differentiable at the origin only if $p > \frac{1}{2}$.
10. Let $f(x,y) = \begin{cases} x^2\tan^{-1}\frac{y}{x} - y^2\tan^{-1}\frac{x}{y}, & xy \neq 0 \\ 0 & ,xy = 0. \end{cases}$ Show that $f_{xy} \neq f_{yx}$ at the origin.
11. Show that $\vec{F} = (2xy+z^3)\hat{\imath} + x^2\hat{\jmath} + 3xz^2\hat{k}$ is a conservative force field. Find the scalar potential. Find the work done in moving an object in this field from $(1,-2,1)\ to\ (3,1,4)$.
12. If $\emptyset(x,y,z) = 2xyz^2, \vec{F} = (xy)\hat{\imath} - z\hat{\jmath} + x^2\hat{k}$ and C is the curve $x = t^2, y = 2t, z = t^3$ from $t = 0\ \ to\ t = 1$. Evaluate (i) $\int \emptyset\, d\vec{r}$ over C. (ii) $\int \vec{F} \times d\vec{r}$ over C.
13. If $\vec{F} = y\hat{\imath} - x\hat{\jmath}$. Evaluate $\int \vec{F}.d\vec{r}$ from $(0,0)\ to\ (1,1)$ along the following paths C
    (i) The Parabola $y = x^2$.
    (ii) The Straight line from $(0,0)\ to\ (1,0) and\ then\ to\ (1,1)$
    (iii) The Straight line joining $(0,0) and\ (1,1)$.
14. Evaluate $\int \frac{-y^3\hat{\imath}+x^3\hat{\jmath}}{(x^2+y^2)^2}.d\vec{r}$ where C is the boundary of the square $x = \pm a, y = \pm a$ in the counter clock wise sense.
15. Find the work done in moving a particle once round a circle C in the xy plane, if the circle has centre at the origin and the radius is 3 units and the force field is given by $\vec{F} = (2x-y+z)\hat{\imath} + (x+y-z^2)\hat{\jmath} + (3x-2y+4z)\hat{k}$.
16. Find the circulation of $\vec{F}$ round the curve C where $\vec{F} = (e^x\sin y)\hat{\imath} + (e^x\cos y)\hat{\jmath}$ and C is the rectangle whose vertices are $(0,0), (1,0), \left(1,\frac{\pi}{2}\right), \left(0,\frac{\pi}{2}\right)$.
17. If $\vec{F} = \vec{\nabla}\emptyset$, where $\emptyset$ is a single valued and has continuous partial derivatives. Show that the work done in moving a particle from one point $P_1 = (x_1, y_1, z_1)$ in this field to another point $P_2 = (x_2, y_2, z_2)$ is independent of the path joining

the two points and conversely if $\int \vec{F}.d\vec{r}$ over C is independent of the path C joining any two points show that there exists a function $\emptyset$ such that $\vec{F} = \vec{\nabla}\emptyset$.

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References:

1. Advanced Differential Calculus of Several Variables by Subir Kumar Mukherjee.
2. Advanced Topics in differential Calculus of Several Variables by Sk Anarul Islam.
3. Vector Analysis by Murray R Spiegel.
4. Vector Calculus by M.D.Raisinghania , H.C.Saxena, H.K.Dass.