Department of Mathematics
Jhargram Raj College
Class: Mathematics(H) Sem-I

Topic: Geometry(2D & 3D) **Date: 31.10.19**

1. If the perpendicular straight lines $ax + by + c = 0$ & $bx - ay + c' = 0$ be taken as the axes of x & y respectively, then show that the equation $(ax + by + c)^2 - 2(bx - ay + c')^2 = 1$ will be transformed into $(y')^2 - 2(x')^2 = \frac{1}{a^2+b^2}$.
2. If by a rotation of rectangular axes about the origin $(ax + by)$ & $(cx + dy)$ be changed to $(a'x' + b'y')$ & $(c'x' + d'y')$ respectively, then show that $ad - bc = a'd' - b'c'$.
3. Show that there is one point whose coordinates do not alter due to a rigid motion.
4. Classify the following equations accordingly to the presence of centre(s
   $(i)\ 2x^2 - 3xy + 5y^2 - 2x + y - 3 = 0$, $(ii)\ 2x^2 + 4xy + 2y^2 - x - y + 5 = 0$,
   $(iii)\ (x - 2y)^2 + 3(x - 2y) + k = 0, k = constant$.
5. Reduce the equation $3(x^2 + y^2) = 2xy = 4\sqrt{2}(x + y)$ to its canonical form. Name the conic and determine the equations of its axes and directrix.
6. Reducing the equation $4x^2 + 4xy + y^2 - 4x - 2y + a = 0$ to its canonical form, determine the nature of the conic for different values of a.
7. Show that the conic represented by $(a^2 + 1)x^2 + 2(a + b)xy + (b^2 + 1)y^2 = c, c > 0$ is an ellipse of area $\frac{\pi c}{|ab-1|}$, $ab \neq 1$.
8. If P and Q are two points on a given conic with focus at S such that $< PSQ$ is a constant, prove that the locus of the point of intersection of tangents at P and Q is also a conic whose focus at S. If the given conic is a parabola and if the tangents at P and Q meet at T, show that $SP.SQ = (ST)^2$.
9. If a focal chord of the conic $\frac{l}{r} = 1 + e\cos\theta$ makes an angle α with the axis, show that the angle between the tangent at its extremities is $\tan^{-1}\frac{2e\sin\alpha}{|1-e^2|}$.
10. A conic $\frac{l}{r} = 1 + e\cos\theta$ is cut by a circle passing through the pole in four points whose radius vectors are $r_i, i = 1(1)4$. Show that $r^{-1} + r^{-2} + r^{-3} + r^{-4} = \frac{2}{l}$ & $\prod_{i=1}^{4} r_i = \frac{4d^2l^2}{e^2}$.
11. Prove that the locus of the midpoint of any focal chord of the conic $\frac{l}{r} = 1 + e\cos\theta$ is $r(1 - e^2cos^2\theta) + el\cos\theta = 0$.
12. Show that the length of the focal chord of the conic $\frac{l}{r} = 1 - e\cos\theta$ which is included to the initial line at angle α is $\frac{2l}{1-e^2cos^2\alpha}$.
13. A Chord PQ of a conic with eccentricity e and semi latus rectum l subtends a right angle at a focus S. Show that $(\frac{1}{SP} - \frac{1}{l})^2 + (\frac{1}{SQ} - \frac{1}{l})^2 = \frac{e^2}{l^2}$.
14. On the ellipse $r = \frac{2l}{5-2\cos\theta}$ find the point with greatest radius vector.
15. If the normals at the points with vectorial angles $\alpha, \beta, \gamma, \delta$ on the conic $\frac{l}{r} = 1 + e\cos\theta$ meet at a point, show that $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}\tan\frac{\delta}{2} + (\frac{1-e}{1+e})^2 = 0$.

16. A variable sphere passes through the points $(0,0,\pm c)$ and cuts the straight lines $y = x\tan\alpha, z = c$ *and* $y = -x\tan\alpha, z = -c$ at the points P and $P^/$ other than $(0,0,\pm c)$. If $PP^/ = 2a$ show that the centre of the sphere lies on the circle $x^2 + y^2 = (a^2 -$ *c2cosec22α, z=0.*
17. If every plane cuts a quadric surface in a circle, show that the surface is sphere.
18. Find the equation of the sphere which passes through the circle $y = 0, (x-a)^2 + (z-c)^2 = r^2$ and touches the plane $x = 0$. Show that the area which it cuts off from the plane $z = 0$ is $\pi(a^2 - c^2)$.
19. The Section of the cone whose guiding curve is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$ by the plane $x = 0$ is a rectangular hyperbola. Show that the locus of the vertex of the cone is the surface $\frac{x^2}{a^2} + \frac{y^2+z^2}{b^2} = 1$.
20. Two cones are described with guiding curves $zx = a^2, y = 0$ & $yz = b^2, x = 0$ and with the same vertex. Prove that if their four common generators meet the plane $z = 0$ in four con cyclic points then the vertex lies on the surface $z(x^2 + y^2) = a^2x + b^2y$.
21. Show that the equation of the cylinder whose generators intersect the curve $ax^2 + by^2 + cz^2 = 1, lx + my + nz = p$ and are parallel to z-axis is $(an^2 + cl^2)x^2 + (bn^2 + cm^2)y^2 + 2lmcxy - 2mcpy - 2lpcx + cp^2 - n^2 = 0$.
22. Show that the perpendiculars from the origin to the generators of the hyperboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ lie on the surface $\frac{a^2(b^2+c^2)^2}{x^2} + \frac{b^2(a^2+c^2)^2}{y^2} = \frac{c^2(a^2-b^2)^2}{z^2}$.
23. Show that the perpendiculars from the origin to the generators of the hyperbolic paraboloid $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z$ lie on the cone $\left(\frac{x}{a} - \frac{y}{b}\right)(ax - by) + 2z^2 = 0$.
24. If the generator through a point P on the hyperboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ meets the principal elliptic section in two points such that the eccentric angle of one is three times that of the other, prove that point P lies on the curve of intersection of the hyperboloid with the cylinder $y^2(z^2 + c^2) = 4b^2z^2$.
25. Prove that the area of the section of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ by the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ is $\frac{2\pi}{3\sqrt{3}}(\sqrt{b^2c^2 + c^2a^2 + a^2b^2})$.

References: 1. Advanced Analytical Geometry by J.G.Chakravorty and P.R.Ghosh.
2. Analytical Geometry including vector analysis.

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