# Department of Mathematics
# Jhargram Raj College
### Class: Mathematics(H) Sem-III

**Topic: C5 (All Units)** **Date: 31.10.19**

1. Show that $\lim_{x\to 0}\frac{x-|x|}{x}$ does not exist.
2. Find $\lim_{x\to 0}\frac{e^{\frac{1}{x}}}{e^{\frac{1}{x}}+1}$.
3. Show that the function $f(x)=x\frac{e^{\frac{1}{x}}-e^{\frac{-1}{x}}}{e^{\frac{1}{x}}+e^{\frac{-1}{x}}}, x\neq 0$ & $f(0)=0$ is continuous at 0.
4. Show that the function $f(x)=\frac{e^{\frac{1}{x^2}}}{1-e^{\frac{1}{x^2}}}, x\neq 0$ & $f(0)=0$ is discontinuous at 0.
5. Let $f(x)=x, -1\leq x\leq 1$ & $f(x+2)=f(x)\ \forall\, x\in\mathbb{R}$. Show that f is discontinuous at every odd integer.
6. Suppose the function $f:\mathbb{R}\to\mathbb{R}$ has limit L at 0 and $a>0$. If $g:\mathbb{R}\to\mathbb{R}$ is defined by $g(x)=f(ax), x\in\mathbb{R}$, show that $\lim_{x\to 0} g(x)=L$.
7. Let $c\in\mathbb{R}$ & $f:\mathbb{R}\to\mathbb{R}\ s.t. \lim_{x\to c}(f(x))^2=L$. Show that if L=0 then $\lim_{x\to c} f(x)=0$. Show by an example that if $L\neq 0$ then f may have no limit at c.
8. Let h: $\mathbb{R}\to\mathbb{R}$ be continuous on $\mathbb{R}$ such that $h\left(\frac{m}{2^n}\right)=0\ \forall\, m\in\mathbb{Z}, n\in\mathbb{N}$. Show that $h(x)=0\ \forall\, x\in\mathbb{R}$.
9. $f:\mathbb{R}\to\mathbb{R}$ such that $f\left(\frac{x+y}{2}\right)=\frac{1}{2}\big(f(x)+f(y)\big)\ \forall\, x,y\in\mathbb{R}$. Show that $f(x)=kx+a$, for some k and a.
10. Let a,b >1 and let f be a bounded function on [0,1] such that f(ax)=bf(x) $0\leq x\leq\frac{1}{a}$. Show that f is continuous at 0.
11. $f:\mathbb{R}\to\mathbb{R}$ be continuous and periodic with period 1. Show that (i) f is bounded above and below and achieves its maximum and minimum (ii) $\exists\, x_0\in\mathbb{R}$ such that $f(x_0+\pi)=f(x_0)$ (iii) f is uniformly continuous on $\mathbb{R}$ .
12. Assume that $f:\mathbb{R}\to\mathbb{R}$ satisfies $f\big(f(x)\big)=-x\ \forall\, x\in\mathbb{R}$. Show that f can't be continuous on $\mathbb{R}$
13. Let $f:[a,b]\to\mathbb{R}$ be continuous in [a,b], differentiable on (a,b). Show that $\exists\,\theta\in(a,b)$ such that $\frac{f'(\theta)}{f(\theta)}=\frac{1}{a-\theta}+\frac{1}{b-\theta}$.
14. Show that the equation $x\ln x=3-x$ has at least one root in (1,3).
15. Let f be differentiable on $\mathbb{R}$ with $a=\sup\{|f'(x)|: x\in\mathbb{R}\}<1$ . Select $s_0\in\mathbb{R}$ and define $s_n=f(s_{n-1}), n\geq 1$. Prove that $\{s_n\}$ is a convergent sequence.


**References: 1. Elements of Real Analysis by Shanti Narayan.**
**2. Introduction to Real Analysis by Robert Bartle and D.Sherbert.**
**3. First Course in Real Analysis by Subir Kr. Mukherjee.**


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