Total Pages-10 B.Sc.-CBCS/IS/MATH/H/C1T/17

2017

MATHEMATICS

[Honours]

(CBCS)

[First Semester]

PAPER - CIT

Full Marks: 60

Time: 3 hours

The figures in the right hand margin indicate marks

UNIT - I

(Calculus - I)

1. Answer any three questions:

 2×3

- (a) What do you mean by asymptote? Does asymptote exist for every curve?
- (b) Your answer find the value of

 $y = \lim_{x \to 1} x^{\frac{1}{x-1}}$ $y = \lim_{x \to 1} x^{\frac{1}{x-1}}$

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- (c) Define point of inflection of a curve.
- (d) Find the envolpe of the straight line

$$\frac{x}{a} + \frac{y}{b} = 1$$

where the parameters a and b are connected by the relation $ab = c^2$.

- (e) Write the Leibnitz theorem successive derivatives up to 4th order.
- 2. Answer any one question:

10 × 1

(a) (i) If s be the length of the arc $3ay^2 = x(x-a)^2$ (b) The arc $3ay^2 = x(x-a)^2$ (c) The arc $3ay^2 = x(x-a)^2$ (d) (i) If s be the length of the arc $3ay^2 = x(x-a)^2$ (e) The arc $3ay^2 = x(x-a)^2$ (f) The arc $3ay^2 = x(x-a)^2$ (g) The a

$$3s^2 = 4x^2 + 3y^2.$$

(ii) Show that the curve

$$y = 3x^5 - 40x^3 + 3x - 20$$

is concave upwards for -2 < x < 0 and $2 < x < \infty$ but convex upwards for $-\infty < x < -2$ and 0 < x < 2. Also show that x = -2, 0, 2 are its points of infexion.

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(b) (i) Trace the curve:

$$x^2y^2 = a(y^2 - x^2)$$
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(ii) If α , β be the roots of the equation $ax^2 + bx + c = 0$ then show that

$$\lim_{x \to \alpha} \frac{1 - \cos(\alpha x^2 + bx + c)}{(x - \alpha)^2} = \frac{1}{2} a^2 (\alpha - \beta)^2$$
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(Calculus - II)

Answer any two questions :

 2×2

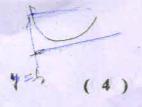
(a) If

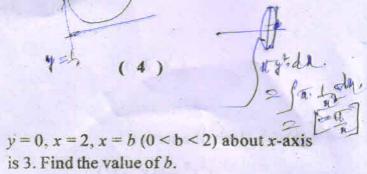
$$I_n = \int_0^{\pi/4} \tan^n x dx,$$

n being a positive integer greater than 1, then prove that

$$I_n = \frac{1}{n-1} - I_{n-2}$$

(b) The volume of the solid generated by the revolution of the curve $y = \frac{1}{x}$, bounded by





is 3. Find the value of b.

- (c) What is the formula for finding area of the curve $y = \psi(t)$, $x = \phi(t)$, where t is the parameter?
- Answer any two questions: = (9(1) 4(1) 4(1)
 - (a) If

$$I_{m,n} = \int_0^{\pi/2} \cos m_x \sin nx \, dx,$$

then show that

$$I_{m,n} = \frac{1}{m+n} + \frac{1}{m+n} I_{m-1, n-1}.$$

Also, deduce that

$$I_{m,n} = \frac{1}{2^{m+1}} \left[2 + \frac{2^2}{2} + \frac{2^3}{3} + \dots + \frac{2^m}{m} \right].$$

(b) Find the area of the surface generated by y dn revolving the curves $x = \cos t$, $y = 2 + \sin t$, $0 \le t \le 2\pi$ about x-axis.

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(y=2) = 1-22 (Configured) 1. 22 (y=2) = 1.

(c) Find the arc length parameter along the curve

$$C: \vec{r}(t) = (1+2t)\hat{i} + (1+3t)\hat{j} + 6(1-t)\hat{k}$$

from the point, where $t = 0$.

UNIT - III
(Geometry)

5. Answer any three questions:

2×3

(a) Determine the type of the conic

$$8x^2 + 10xy + 3y^2 + 22x + 14y + 15 = 0$$

(b) Show the plane z-1=0 which intersects the ellipsoid

$$\frac{x^2}{48} + \frac{y^2}{12} + \frac{z^2}{4} = 1$$

is an ellipse. Determine semi-axes.

(c) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 25$, x + 2y - z + 2 = 0 and the point (1, 1, 1).

- (d) Find the equation of the cylinder where generators are parallel to the line $x = -\frac{y}{2} = \frac{z}{3}$ and whose guiding curve is the ellipse $x^2 + y^2 = 1$, z = 3.
- (e) Find the equation of a right circular cone whose axis is $\frac{x}{1} = \frac{y}{0} = \frac{z}{-2}$ and radius equal to 7.
- 6. Answer any one question:

5 × 1

- (a) Find the polar equation of the tangent to the circle $r = 2d \cos\theta$ at the point whose vectorial angle is θ_1 .
- (b) Find the equations of the generating lines of the hyperboloid of one sheet

$$\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = 1$$

which passes through the point (2, 3, -4).

7. Answer any one question:

 10×1

- (a) (i) Find the locus of a luminous point, if the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ casts a circular shadow on the plane z = 0
 - (ii) Reduce the equation

$$4x^2 - 4xy + y^2 - 8x - 6y + 5 = 0$$

to canonial form.

- (b) (i) Prove that the locus of the foot of the perpendecular from a focus of the conic $\frac{l}{r} = 1 e \cos \theta \text{ on a tangent to it, is given by}$ $r^{2}(1-e^{2}) 2ler \cos \theta l^{2} = 0$
 - (ii) Prove that the axes of sections of the conicoid $ax^2 + by^2 + cz^2 = 1$ which pass through the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ lie on the cone

$$\frac{(b-c)}{x}(mz-ny) + \frac{(c-a)}{y}(nx-lz) + \frac{(a-b)}{z}(ly-mx) = 0. \quad 5$$

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UNIT - IV

(Differential Equation)

8. Answer any two questions:

 2×2

 (a) Find the integrating factor of the following differential equation

$$\frac{dx}{dy} + \frac{xy}{1 - y^2} - y\sqrt{x} = 0$$

(b) Reduce the equation

$$\sin y \frac{dy}{dx} = \cos x (2\cos y - \sin^2 x)$$

into a linear equation.

(c) Show that the equation

$$M(x, y)dx + N(x, y)dy = 0$$

will be exact if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial y}$$

9. Answer any one question:

5 × 1

(a) (i) Show that the substitution z = ax + by + c changes

4.

$$y' = f(ax + by + c)$$

into an equation with separable variables, and apply this method to solve the equation

$$y' = \sin^2(x - y + 1)$$

(ii) Reduce the equation

$$(2x^{2}-1)\left(\frac{dy}{dx}\right)^{2} + (x^{2}+y^{2}+2xy+2)\frac{dy}{dx} + 2y^{2}+1=0$$

to Clairaut's form by the substitution x+y=u and xy-1=v, hence solve the equation.

(b) (i) Show that if y_1 and y_2 be solutions of the equation

$$\frac{dy}{dx} + Py = Q$$

where P and Q are functions of x alone,

and
$$y_2 = y_1 z$$
 then $z = 1 + ae^{-\int_{y_1}^{Q} dx}$

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(ii) Show that the solution of $\frac{dy}{dx} + py = Q$ can also be written in the form

$$y = \frac{Q}{P} - e^{\int P dx} \left[c + \int e^{\int P dx} d\left(\frac{Q}{P}\right) \right]$$
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