বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examinations 2020

(Under CBCS Pattern)

Semester - I

Subject: MATHEMATICS

Paper: C 1-T

Full Marks : 60

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Answer any ***three*** from the following questions : 3×20

1. (a) Evaluate the following limits : $\lim\limits_{x\to 0} x ln(\sin x)$ in $(0,\pi)$. 4

(b) Show that the four asymptotes of the curve

$(x^2-y^2)(y^2-4x^2)+6x^3-5x^2y-3xy^3+2y^3-x^2+3xy-1=0$ cut the curve in eight points which lie on the circle $x^2+y^2=1$. 6

(c) Prove that the envelope of a variable circle whose centre lies on the parabola $y^2=4ax$ and which passes through its vertex is $2ay^2+x(x^2+y^2)=0$ 6

(d) What are the points of inflection of the function $f(x)=3x^4-8x^3$. 4

2. (a) What do you mean by rectillinear asymptotes to a curve ? 4

(b) Find the equation of the envelope of the family of curve represented by equation $x^2\sin\alpha+y^2\cos\alpha=a^2$. 4

(c) If $y=\left(\sin^{-1}x\right)^2$ show that $\left(1-x^2\right)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$. Also find $y_n(0)$. 6

(d) Find the asymptotes of the curve $(x+y)(x-2y)(x-y)^2+3xy(x-y)+x^2+y^2=0$. 6

3. (a) If $I_n=\int_0^1 x^n\tan^{-1}x\,dx$, $n>2$ then prove that $(n+1)I_n+(n-1)I_{n-2}+\frac{1}{n}=\frac{\pi}{2}$. 4

(b) Determine the length of one arc of the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$. 4

(c) Find the reduction formula for $\int\sin^m xCos^n xdx$ where either m or n or both are negative integers. And hence find $\int\frac{\cos^4 x}{\sin^2 x}dx$. 6

(d) Find the whole length of the loop of the curve $9ay^2=(x-2a)(x-5a)^2$. 6

4. (a) Find the eccentricity and the vertex of the conic $r=3\sec^2\frac{\theta}{2}$. 4

(b) Find the polar equation of the ellipse $\frac{x^2}{36}+\frac{y^2}{20}=1$. 4

(c) A sphere of radius k passes through the origin and meets the axes in A, B, C. Prove that the locus of the centroid of the triangle ABC is the sphere $9\left(x^2+y^2+z^2\right)=4k^2$. 6

(d) Show that the plane $y+6=0$ intersects the hyperbolic paraboloid $\frac{x^2}{5}-\frac{y^2}{4}=6z$ in parabola. 6

5. (a) For what angle must t he axes be turned to remove the term x^2 from $x^2-4xy+3y^2=0$. 4

(b) Find the centre and the radius of the circle $3x^2+3y^2+3z^2+x-5y-2=0$, $x+y=2$. 4

(c) P is a variable point such that its distance from the xy-plane is always equal to one fourth the square of its distance from the y-axis. Show that the locus of P is a cylinder. 6

(d) Reduce the equation $7x^2+y^2+z^2+16yz+8zx-8xy+2x+4y-40z-14=0$ to the canonical form and find the nature of the conicoid it represents. 6

6. (a) Solve : $\left(1+y^2\right)dx-\left(\tan^{-1}y-x\right)dy=0$. 4

(b) Find the singular solution of $xp^2-\left(y-x\right)p-y=1$. 4

(c) Solve and find the singular solutions of $p^4=4y\left(xp-2y\right)^2$. 6

(d) Solve : $y\left(xy+2x^2y^2\right)dx+x\left(xy-x^2y^2\right)dy=0$. 6