বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examinations 2020

(Under CBCS Pattern)

Semester - I

Subject: MATHEMATICS

Paper: GE 1-T

Full Marks : 60

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

Answer any ***three*** from the following questions : 3×20

1. (a) Show that the curve $y^3 = 8x^2$ is Concave to the foot of the ordinate everywhere except at Origin. 4

   (b) State some natures of Hyperbolic Sine. 4

   (c) If $y = 2\cos x(\sin x - \cos x)$, show that $y_{10}(o) = 2^{10}$. 6

   (d) Find the envelopes of the straight line $\frac{x}{a}+\frac{y}{b}=1$ where the parameters a and b are connected by the relation $a^2+b^2=c^2$ 6

2. (a) If $y=(ax+b)^m$ find $D^n(ax+b)^m$. 4

(b) Evaluate $\underset{x\to 0}{Lt}(\cos mx)^{\frac{n}{x^2}}$. 4

(c) Find the length of a quadrant of the circle $r=2a\sin\theta$. 4

(d) Evaluate $\int_0^{\pi/2}\sin^8 x\cos^6 xdx$. 4

(e) The circle $x^2+y^2=a^2$ revolves about the x-axis. Show that the surface area and the volume of the sphere thus generated are respectively $4\pi a^2$ and $\frac{4}{3}\pi a^3$. 4

3. (a) Evaluate $\int_0^{\pi/4}\tan^5 xdx$. 4

(b) Find the volume of the solid generated by revolving the part of parabola $x^2=4ay, a>0$ between the ordinates $y=0$ and $y=a$ about its axis. 4

(c) Find the area of the smaller portion enclosed by the curves $x^2+y^2=9$ and $y^2=8x$. 6

(d) Trace out the curve cycloid

$x=a(\theta-\sin\theta),\ y=a(1-\cos\theta)$ 6

4. (a) Through what angle must be the axis be turned to remove xy term from $7x^2+4xy+3y^2=0$. 4

(b) If pair of lines $x^2-2pxy-y^2=0$ and $x^2-2qxy-y^2=0$ be such that each pair bisects the angles between the other pair, prove that $pq+1=0$. 4

(c) Find the equaiton of the cylinder whose generators are parallel to the straight line $\frac{x}{-1}=\frac{y}{2}=\frac{z}{3}$ and whose guiding curve is $x^2+y^2=9, z=1$. 6

(d) The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the co-ordinate axes *A*, *B*, *C*. Fine the equation of the cone generated by the straight lines drawn from 0 to meet the circle *ABC*. 6

5. (a) Show that the semi-latus rectum of a conic is the harmonic mean between the segments of a focal chord. 4

(b) Find the equation of the circle on the sphere $x^2+y^2+z^2=49$ whose centre is at the point $(2,-1,3)$. 4

(c) Show that the straight line $r\cos(\theta-\alpha)=p$ touches the conic $\frac{l}{r}=1+e\cos\theta$ if $(l\cos\alpha-ep)^2+l^2\sin^2\alpha=p^2$. 6

(d) Find the equation of the plane which passes through the point $(2,1,-1)$ and is orthogonal to each of the planes $x-y+z=1$ and $3x+4y-2z=0$. 6

6. (a) Find t he differential equation of all circles passing through the origin having centres on the $x-axis$. 4

(b) Find an integrating factor of the differential equation

$$\left(3x^2y^4+2xy\right)dx+\left(2x^3y^3-x^2\right)dy=0$$ 4

(c) Find the general and the singular solutions of $y=px+\sqrt{a^2p^2+b^2}$. 6

(d) Reduce the differential equation $\left(px^2+y^2\right)\left(px+y\right)=\left(p+1\right)^2$ to clairaut's form by the substitution $u=xy, v=x+y$ and then find the general solution. Where $p=\frac{dy}{dx}$. 6
