বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examinations 2020

(Under CBCS Pattern)

Semester - V

Subject: MATHEMATICS

Paper: C11T

(Partial Differential Equations & Applications)

Full Marks : 60

Time : 3 Hours

Candiates are required to give their answer in their own words as far as practicable.

The figures in the margin indicate full marks.

Attempt any ***three*** questions. 3×20=60

1. (a) Find the integral surface passing through the curve $y^2+z^2=1$, $x+z=2$ and corresponding to the PDE $4yzp+q=-2y$. 10

(b) (i) Find PDE corresponding to the equation $z=xy+f\left(x^2+y^2\right)$, f being an arbitrary function.

(ii) Find the PDE of the family of right circular cone whose axis coincides with z axis. 5+5

2. (a) Reduce the PDE $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$ to $\frac{\partial^2 z}{\partial u \partial v} = 0$ by $u = x - ct$, $v = x + ct$. 10

(b) (i) Solve the PDE by Lagrange's method $py + qx = xyz^2\left(x^2 - y^2\right)$. 5

(ii) Solve the PDE $px + qy = z\left(1 + pq\right)^{1/2}$. 5

3. (a) Solve the following one dimensional heat equation

$$\frac{\partial T}{\partial t} - k\frac{\partial^2 T}{\partial x^2} = 0, 0 \le x \le l, t \ge 0$$

Subject to the condition

(i) $T(x, 0) = f(x) = l - x,\ 0 \le x \le l$

(ii) $T(0,\ t) = T(l,\ t) = 0,\ t \ge 0$

(iii) $T(x, t) < \infty$ as $t \to \infty$.

Hence evaluate $\lim\limits_{t \to \infty} T(x,\ t)$.

where k is a constant. 15

(b) Find the general solution of the PDE $x\left(y^n - z^n\right)p + y\left(z^n - x^n\right)q = z\left(x^n - y^n\right)$. 5

4. (a) Find the solution of the following two-dimensional Laplace Equation at any interior of the rectangle $0 \le x \le a,\ 0 \le y \le b$, $\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0$, subject to the boundary conditions

$\varphi_x(0,\ y) = \varphi_y(a,\ y) = 0,\ 0 \le y \le b$

and $\varphi_y(x,\ 0) = 0$; $\varphi_y(x,\ b) = f(x),\ 0 \le x \le a$. 15

(b) Find the complete integral of the PDE $z^2 = \frac{\partial z}{\partial x}\frac{\partial z}{\partial y}xy$, by Charpit's method. 5

5. (a) Solve the following equation for a string of finite length $u_{tt} - 9u_{xx} = 0,\ 0 \le x \le 2,\ t \ge 0$. Subject to the boundary conditions $u(0,\ t) = u_t(0,\ t) = 0,\ u(2,\ t) = u_t(2,\ t) = 0,\ t \ge 0$ and the initial condition $u(x,\ 0) = x,\ u_t(x,\ 0) = 0,\ 0 \le x \le 2$. 15

(b) The general solution of the equation $\left(D^2 - 2DD' + D'^2\right)u = e^{x+2y}$. 5

6. (a) Solve the one dimensional wave equation of infinite string

$$u_{tt} - c^2 u_{xx} = 0,\ 0 \le x \le \infty,\ t \ge 0$$

subuject to the initial coditions $u(x,\ 0) = f(x),\ u_t(x,\ 0) = g(x),\ x \ge 0$ and the boundary condition $u(0,\ t) = 0,\ t \ge 0$. 15

(b) Find the P.I of the equation $(D - D')^2 z = \tan(x + y)$. 5