বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examinations 2021

(Under CBCS Pattern)

Semester - V

Subject: MATHEMATICS

Paper: DSE2T

Full Marks : 60

Time : 3 Hours

Candiates are required to give their answer in their own words as far as practicable.

The figures in the margin indicate full marks.

PROBABILITY AND STATISTICS

Answer any ***three*** questions. $3\times20=60$

1. (a) Show that Poisson approximation is a limiting case of Binomial law. 6

(b) Two numbers x and y are selected at random from the set of number {1, 2, 3, 4, ...99). Find the probability $P\left(|x-y|\geq 54\right)$. 8

(c) Let X be a standard normal variate then find the distribution of $Y=\frac{1}{2}X^2$. 6

2. (a) Prove that for any random variable X (discrete or continuous) and for any real number $cE(|X-c|) \geq E(|X-\mu|)$, Provided the expectations exists and μ is the medial of X. 7

(b) Let X be a random variable having Poisson distribution with parameter μ and the conditional distribution of Y given $X=i$ be given by $f_{i,j} = \binom{i}{j} p^i q^j$ for $0 \leq j \leq i$, $i \neq 0$, $p+q=1$. Find the marginal distribution of Y. 6

(c) If $f(x, y) = \begin{cases} \frac{6-x-y}{8}, 0<x<2,\ 2<y<4 \\ 0, \quad \text{elsewhere} \end{cases}$, find $P(X+Y<3)$. 7

3. (a) The jdf (joint density function) of X and Y is given by

$$f(x, y) = \begin{cases} k(x+y),\ 0<x<10<y<1 \\ 0, \quad \text{elsewhere} \end{cases}$$

Find find $P(|X-Y|) \leq 1/2$ and $f_X(x)$ and $f_Y(y)$. Are X and Y independent ? 7

(b) Let X and Y be dindependent random variable having the normal density $(0, \sigma)$. Find $P(x^2+y^2 \leq 1)$. 6

(c) The joint probability density function of the random variable X and Y is

$$f(x,\ y) = \begin{cases} k(1-x-y),\ x \geq 0,\ y \geq 0,\ x+y \leq 1 \\ 0, \quad \text{elsewhere} \end{cases},$$

where k is a constant. Find mean value of Y when $X=1/2$ and the covariance of X and Y. 7

4. (a) If X and Y are connected by $2X+3Y+4=0$, then show that $\rho(X,\ Y)=-1$. 6

(b) Let the joint probability density function of X and Y be given by $f(x,\ y) = x^2 + \frac{xy}{3}, 0, x, 1, 0, y, 2 : 0$ elsewhere. Find regression line of x on y. 7

(c) If X and Y are two independent random variable having the density function respectively

$$f_X(x) = \begin{cases} e^{-x}, & x > 0 \\ 0 & \text{elsewhere} \end{cases} \quad \text{and} \quad f_Y(y) = \begin{cases} e^{-y}, & y > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Find the density function of $\dfrac{X}{X+Y}$. 7

5. (a) Show by Chebyshev's inequality that 2000 throws with a coin the probability that the number of heads lies between 900 to 1100 is 19/20. 6

(b) A random variable X has probability density function $12x^2(1-x)$, $(0 < x < 1)$. Compute $P(|x-m| \geq 2\sigma)$, compare it with the limit given by Chebyshev's inequality. 7

(c) A random sample of 500 apples was taken from a large consignment and 60 were bad. Obtain the 98% confidence limits for the percentage number of bad apples in the consignment. 7

6. (a) Sample of two types of electric light bulb were tested for length of life and the following data were obtained:

	Type-I	Type-II
Sample no	$n_1 = 8$	$n_2 = 7$
Sample means	$\bar{x}_1 = 1234$ hrs	$\bar{x}_2 = 1036$ hrs
Sample s.d	$s_1 = 36$ hrs	$s_2 = 40$ hrs

Is the difference in the mean sufficient to warren that the Type I is superior to Type II regarding the length of life.

(b) Obtain the recurrence relation $\mu_{K+1} = \mu\left(K\mu_{K-1} + \dfrac{d\mu_K}{d\mu}\right)$ for the Poisson distribution with parameter μ. Hence, find the coefficient of Skewness and Coefficient of excess of this Poisson distribution. 7

(c) If X is uniformly distribution over $(-1, 1)$, then find the distribution of $|X|$. 6

BOOLEAN ALGEBRA AND AUTOMATA THEORY

Answer any ***three*** questions. 3×20=60

1. (a) Tabulate the Chomsky hierarchy with an example for each type of grammar.

   (b) What are universal logic gate ? Why those are called universal ?

   (c) With a suitable example, explain various asymptotic notations.

   (d) Explain lattice, sublattice, explain with example. 5+5+5+5

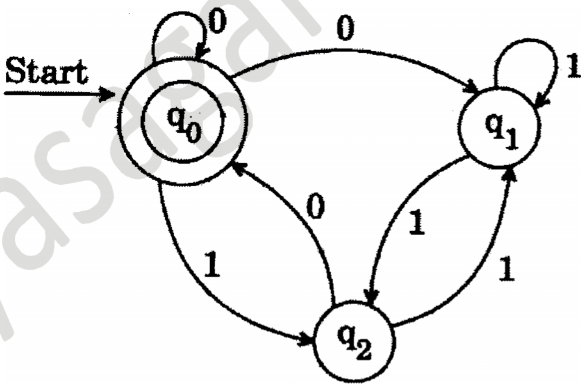
2. Construct a Turing Machine that recognizes the language $L = \{0^{nm} : n,\ m >= 0\}$. 20

3. Reduce the given CFG with Productions given by

   S $\rightarrow$ abSB / a / aAb and

   A $\rightarrow$ bS / aAAb to Chomsky Normal form. 20

4. Deduce R.E. from the Fig. and check whether the string 0100 is accepted or not. 20



5. Define a regular set. Using Pumping Lemma, show that the language

   $L = \{a^n b^k : n > k \text{ and } n >= 1\}$ is not regular. 10+10

6. Among the first 1000 positive integers :

   (a) Determine the integers which are not divisible by 5, nor by 7 nor by 9.

   (b) Determine the integers divisible by 5 but not by 7 not by 9. 10+10=20

PORTFOLIO OPTIMIZATION

Answer any ***three*** questions. 3×20=60

1. Prove that the expected return μ_i on any asset i satisfies $\mu_i = r_f + \beta_i\left(\mu_M - r_f\right)$, $\beta_i = \frac{\sigma_{iM}}{\sigma_{M^2}}$ and σ_{iM} is the covariance of the return on asset i and the market protfolio r_M ; $\sigma_M^2 = \operatorname{var}\left(r_M\right)$.

2. Consider 3 assets with rates of return r_1, r_2 and r_3 respectively. The covariance matrix and expected rates of return are $\Sigma = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$ and $m = \begin{pmatrix} 0.4 \\ 0.4 \\ 0.8 \end{pmatrix}$

   (a) Find the minimum variance portfolio.

   (b) Find a second efficient portfolio.

   (c) If the risk free rate is $r_f = 0.2$, find an efficient portfolio of risky assets.

3. For the Markowitz mean-variance portfolio, solve the quadratic programming problem

   Minimize $\quad \frac{1}{2} w^T \Sigma w - \lambda m^T w$

   Subject to $\quad e^T w = 1$

   where $w = \left(w_1, w_2, \ldots\ldots w_n\right)^T$, $m = \left(m_1, m_2, \ldots\ldots m_n\right)^T$

   $\mu_i = E\left(r_i\right)$, $z = \left(r_1, r_2, \ldots\ldots r_n\right)^T$, $\operatorname{cov}\left(z\right) = \Sigma$

4. Assume that the expected rate of return on the market portfolio is 24% $\left(r_M = 0.24\right)$ and the rate of return on T-Bills (risk free rate) is 7% $\left(r_f = 0.07\right)$. The standard deviation of the market is 33% $\left(\sigma_M = 0.33\right)$. Assume that the market portfolio is efficient.

   (a) What is the equation for the capital market line ?

   (b) If an expected return of 38% is desired, what is the standard deviation of this position?

5. (a) Define (i) Beta of a portfolio

(ii) Security market line

(b) You have a protfolio with a beta of 0.84. What will be the new portfolio beta if you keep 85% of your money in the old portfolio and 14% in a stock with a beta of 1.93 ?

6. (a) What are some of the benefits of diversifiction ?

(b) Use the information in the following to answer the questions below.

State of Economy	Probability of state	Return on A in state	Return on B in state
Boom	35%	0.040	0.210
Normal	50%	0.030	0.080
Recession	15%	0.046	–0.010

(i) What is the expected return of each asset ?

(ii) What is the variance of each asset ?