DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – I , INTERNAL ASSESSMENT-1st , 2017-18
Sub: MATHEMATICS, Course – C1

Full Marks: 10 **Time: 30 m.**

Answer any five questions: $(\mathbf{2 \times 5 = 10})$

01. The envelope of the lines $\frac{x}{a} + \frac{y}{b} = 1$ $(a, b\ are\ variable\ parameters) is\ given\ by$ $\sqrt{x} + \sqrt{y} = \sqrt{k}$ (k is given constant).Find a relation between a and b.

02. The gradient of one of the straight lines of $ax^2 + 2hxy + by^2 = 0$ is twice that of the other. Show that $8h^2 = 9ab$.

03. Prove that $\lim_{x\to 0^+} (\frac{sinx}{x})^{\frac{1}{x}}$ =1.

04. Solve: $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$.

05. Find the translation which transforms the equation $x^2 + y^2 - 2x + 14y + 20 = 0$ into $(x^/)^2 + (y^/)^2 - 30 = 0$.

06. If $y = \sin(m \sin^{-1} x)$, show that (a) (1- $x^2)y_2 - xy_1 + m^2y = 0$ (b) (1- $x^2)y_{n+2} - (2n +$ *1xyn+1+(m2−n2)yn=0.*

07. Let $I_n = \int_0^{\frac{\pi}{2}} sin^n x dx$. Prove that $I_n = \frac{n-1}{n} I_{n-2}$ $(n>1)$. Hence deduce the value of $\int_0^{\frac{\pi}{2}} sin^5 x dx$.

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – I , INTERNAL ASSESSMENT-2nd , 2017-18
Sub: MATHEMATICS, Course – C1

Full Marks: 10 **Time: 30 m.**

Answer any five questions: $(\mathbf{2 \times 5 = 10})$

01. Solve : $(1 + x^2)\frac{dy}{dx} + (1 - x)^2 y = xe^{-x}$.
02. Let $I_{m,n} = \int_0^{\frac{\pi}{2}} sin^m x cos^n x dx$, m, n being a positive integer greater than 1. Prove that $I_{m,n} = \frac{m-1}{m+n} I_{m,n-2} = \frac{m-1}{m+n} I_{m-2,n}$.
03. If $I_n = \int e^{ax} cos^n x dx$, Show that $(a^2 + n^2)I_n = e^{ax} cos^{n-1} x(acosx + nsinx) + n(n-1)I_{n-2}$, n being a positive integer greater than 1.
04. Find the co-ordinates of the centre and the radius of the circle $x - 2y - 2z + 7 = 0, x^2 + y^2 + z^2 - 2x + 6y + 4z - 35 = 0$.
05. Find the equation of the cone whose vertex is (1,0,-1) and which passes through the circle $x^2 + y^2 + z^2 = 4, x + y + z - 1$.
06. Show that the curve $y = x^3$ has a point of inflexion at $x = 0$.
07. Find the Vertical Asymptotes of $y = (a - x)(tan\frac{\pi x}{2a})$

DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc(H) Sem – I , INTERNAL ASSESSMENT-1st , 2018-19

Sub: MATHEMATICS, Course – C1

Full Males: 10 **Time: 30 m.**

Answer any five questions: $(\mathbf{2 \times 5 = 10})$

1. Let $x = \varphi(t)$ and $y = \mu(t)$, show that $\frac{d^2y}{dx^2} = \frac{x_1y_2 - x_2y_1}{x_1^3}$, where the suffixes denote the order of differentiation with respect to t. Hence obtain $\frac{d^2y}{dx^2}$, if $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$.
2. State "**Leibnitz's Theorem on successive derivatives**". Prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$, where $y = \sin(msin^{-1}x)$ and both m and n are natural numbers.
3. Solve : $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$.
4. Solve :$(1 + x^2)\frac{dy}{dx} + (1 - x)^2y = xe^{-x}$.
5. Determine the equation of the Sphere through the circle $x^2 + y^2 + z^2 + 6x - 8y - 4z + 4 = 0, x + 2y + 3z = 6$ and through the point $(2,3,1)$.
6. If under an orthogonal transformation the expression $ax^2 + 2hxy + by^2 = 0$ changes to $AX^2 + 2HXY + BY^2 = 0$ then Show that $a + A = b + B$.
7. Evaluate $\int_0^{\frac{\pi}{2}} sin^4xcox^8\,xdx$, By using Reduction Formula.
8. Evaluate$\int_0^{\frac{\pi}{6}} tan^6\,xdx$, By using Reduction Formula.

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – I , INTERNAL ASSESSMENT-2nd , 2018-19
Sub: MATHEMATICS, Course – C1

Full Marks: 10 **Time: 30 m.**

Answer any five questions: $(\mathbf{2 \times 5 = 10})$

1. Find the point of inflextion, if any, of the curve $x = (\log y)^3$.
2. Find the asymptotes of the curve $x^2y^2 - a^2(x^2 + y^2) - a^3(x + y) + a^4 = 0$. Also determine the nature of the asymptotes as obtained.
3. Find the length of the arc of the parabola $y^2 = 16x$ measured from the vertex to an extremity of the latus rectum.
4. Show that the entire area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab.
5. Show that the polar equation of the straight line passing through the points (r_1, θ_1) and (r_2, θ_2) is $\frac{1}{r}\sin(\theta_1 - \theta_2) - \frac{1}{r_1}\sin(\theta - \theta_2) + \frac{1}{r_2}\sin(\theta - \theta_1) = 0$. Hence find the condition of co-linearity of the points $(r_1, \theta_1), (r_2, \theta_2) and\ (r_3, \theta_3)$.
6. Find the equations of the generators of the hyperboloid $x^2 - y^2 = 4z$ which pass through the point (7,5,6).
7. Find the equation of the right circular cone with the vertex at $(1,2,-1)$, semi vertical angle 60° and the axis being $\frac{x-1}{3} = \frac{y+2}{-4} = \frac{z+1}{5}$.
8. Solve : $\frac{dy}{dx} + 2xy = e^{-x^2}$.

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – I , INTERNAL ASSESSMENT-1st , 2019-20
Sub: MATHEMATICS, Course – C1

Full Marks: 10 **Time: 30 m.**

Answer any five questions: **$(2 \times 5 = 10)$**

1. Starting from $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$ obtain the expressions for $\frac{d^2x}{d^2y}$ & $\frac{d^3x}{d^3y}$.
2. If $y = \frac{x^3}{x^2-1}$, then prove that $(y_n)_0 = \begin{cases} 0, & \text{if } n \text{ is even;} \\ -n!, & \text{if } n \text{ is odd;} \end{cases} n > 1.$
3. Evaluate $\int tan^5 x\, dx$, using reduction formula.
4. Evaluate $\int_0^{\frac{\pi}{2}} sin^{10}\, x\, dx$, using Walli's formula.
5. Show that the spheres $x^2 + y^2 + z^2 - 2x + y - 3z + 4 = 0$ & $x^2 + y^2 + z^2 - 5x - 6y + 2z - 5 = 0$ cut orthogonally.
6. If under an orthogonal transformation the expression $ax^2 + 2hxy + by^2 = 0$ changes to $AX^2 + 2HXY + BY^2 = 0$ then Show that $ab - h^2 = AB - H^2$.
7. Obtain the equation of the sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0, x + y + z = 3$ as the great circle.
8. Solve: $(1 + x^2)\frac{dy}{dx} + (1 - x)^2 y = xe^{-x}$.

DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc(H) Sem – I , INTERNAL ASSESSMENT-2nd , 2019-20

Sub: MATHEMATICS, Course – C1

Full Marks: 10 **Time: 30 m.**

Answer any five questions: $(\mathbf{2 \times 5 = 10})$

1. Prove that, if $a > 0$, then $\lim_{x\to 0+0} \frac{x}{a}\left[\frac{b}{x}\right] = \frac{b}{a}$ and $\lim_{x\to 0+0} \left[\frac{x}{a}\right]\frac{b}{x} = 0$, where $[x]$ is the greatest integer in x but not greater then x. Discuss the left hand limit of these functions.
2. Examine the asymptotes, if any, parallel to the Y – axis of the curve $x^2y^2 - 9x^2 + 2 = 0$
3. Find the length of the circumference of the circle $x^2 + y^2 = 16$.
4. Find the area of the Cardioide $\boldsymbol{r} = \boldsymbol{a}(1 - cos\,\theta)$.
5. Show that the straight line $r\cos(\theta - \alpha) = p$ touches the conic $\frac{l}{r} = 1 + e\cos\theta$ if $(l\cos\alpha - ep)^2 + l^2 sin^2\alpha = p^2$.
6. Find the angle between the lines in which $x - 3y + z = 0$ cuts the cone $x^2 - 5y^2 + z^2 = 0$.
7. Find the equations of the generators of the hyperboloid $x^2 - y^2 = 2z$ which pass through the point $(5,3,8)$.
8. Solve: $\frac{dy}{dx} + \frac{y}{x}\ln y = \frac{y}{x^2}(\ln y)^2$.