DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – I , INTERNAL ASSESSMENT-1st , 2017-18
Sub: MATHEMATICS, Course – C2

Full Marks: 10 **Time: 30 m.**
Answer any five questions: $(\mathbf{2 \times 5 = 10})$

01. Dose there exist square matrices A and B of order 2 such that AB – BA = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$?
02. Prove that in a real vector space V $(-c)v = -(cv)$, where $c \in \mathbb{R}, v \in$ V.
03. Prove that the roots of the equation $(x+4)(x+2)(x-3)+(x+3)(x+1)(x-5)=0$ are all real and different.
04. If a be an integer, Prove that for all positive integers n, gcd(a,a+n) is a divisor of n. Hence prove that gcd(a,a+1) =1.
05. Z is a variable complex Number such that $\left|z - \frac{10}{z}\right|$ = 3.Find the greatest and the least value of $|Z|$.
06. Rank of the matrix A = $\begin{pmatrix} 0 & 0 & 2 & 2 & 0 \\ 1 & 3 & 2 & 4 & 1 \\ 2 & 2 & 2 & 6 & 2 \end{pmatrix}$ is 2 correct or justify.
07. Let $f: A \to B$ and $P \subset A$. Prove that (i) $P \subset f^{-1}(f(P))$ (ii)) $P = f^{-1}(f(P))$ if f is injective.

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE

B.Sc(H) Sem – I , INTERNAL ASSESSMENT-2nd , 2017-18

Sub: MATHEMATICS, Course – C2

Full Marks: 10 **Time: 30 m.**

Answer any five questions: $(\mathbf{2 \times 5 = 10})$

01. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = sinx, x \in \mathbb{R}$.Prove that the function is neither injective nor surjective. Reduce the codomain to T = $\{x \in \mathbb{R}: -1 \leq x \leq 1\}$.Then prove that $f: \mathbb{R} \to T$ define by $f(x) = sinx$ is surjective but not injective.
02. If $a \equiv b(mod\ m)$ then prove that $a^n \equiv b^n(mod\ m)$ for all positive integers n.
03. Solve by cardan's Method $x^3 - 6x - 9 = 0$.
04. If α be a special root of $\alpha^{11} - 1 = 0$, Prove that $(\alpha + 1)(\alpha^2 + 1) \ldots \ldots \ldots (\alpha^3 + 1) = 1$.
05. $T: \mathbb{R}^3 \to \mathbb{R}^3$ is defined as $T(x, y, z) = (x + 2y + 3z, 2x + 3y + z, 3x + y + 2z)$. Find the Matrix of T relative to Ordered basis $\{(-1,1,1), (1,-1,1), (1,1,-1)\}$ of$\mathbb{R}^3$.
06. β be an eigen value of a real Skew-Symmetric Matrix. Prove that $\frac{|1-\beta|}{|1+\beta|} = 1$.
07. Let V and W be Vector Spaces over R. Let T:V→W be a linear mapping then T is one-one iff ker T=$\{\boldsymbol{\theta}\}$.

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – I , INTERNAL ASSESSMENT-1st , 2018-19
Sub: MATHEMATICS, Course – C2

Full Marks: 10 **Time: 30 m.**
Answer any five questions: $(\mathbf{2 \times 5 = 10})$

1. Define modulus of a complex number. Prove that the modulus of the product of two complex numbers is the product of their moduli.
2. (a) State and prove the conditions of equality for the following –
$$|z_1 + z_2| \leq |z_1| + |z_2|$$
(b) Write down the polar representation of the complex number $(0,1)$.
3. Find the least positive value of the expression $36x + 40y, x, y \in \mathbb{Z}$.
4. Examine if the relation ρ on the set S is (i) reflexive (ii) symmetric (iii) transitive
S is the set of all lines on a plane and ρ is defined on S by " $l \rho m$ iff l is perpendicular to m" for $l, m \in S$.
5. Determine the conditions for which the system of equation has (a) only one solution (b) many solutions.
$x + 2y + z = 1$
$$2x + y + 3z = b$$
$$x + ay + 3z = b + 1$$
6. For what values of k the planes $x + y + z = 2, 3x + y - 2z = k, 2x + 4y + 7z = k + 1$; form a triangular prisom.
7. Prove that the set of vectors $\{(1,2,3), (2,3,1), (3,1,2)\}$ are linearly independent in $\mathbb{R}^3$.
8. Find a basis for the Real Vector Space $\mathbb{R}^3$, that contains the vectors (1,0,1) and (1,1,1).

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – I , INTERNAL ASSESSMENT-2nd , 2018-19
Sub: MATHEMATICS, Course – C2

Full Marks: 10 **Time: 30 m.**

Answer any five questions: $(\mathbf{2 \times 5 = 10})$

1. Prove that the intersection of two subspace of a vector space V over the field F is a subspace of V.
2. Find the dimension of the subspace S of $\mathbb{R}^3$ defined by S = $\{(x, y, z) \in \mathbb{R}^3 : 2x + y - z = 0\}$
3. If $p \; and \; q$ are distinct primes and a is any integer, prove that $a^{pq} - a^p - a^q + a$ is divisible by pq.
4. Let $f: A \to B \; and \; P \subset A$. Prove that $P \subset f^{-1}f(P) \; and \; P = f^{-1}f(P)$ if f be injective.
5. T:$\mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x + 1, y + 1, z + 1), (x, y, z) \in \mathbb{R}^3$ is a linear transformation?
6. Find the Eigen values of the matrix $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{pmatrix}$.
7. If $\cos\alpha + \cos\beta + \cos\gamma = 0$ and $\sin\alpha + \sin\beta + \sin\gamma = 0$, prove that
   a. $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$
   b. $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$
8. Apply Descartes' Rule of signs to ascertain the minimum number of complex roots of the equation $x^6 - 3x^2 - 2x - 3 = 0$.

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – I , INTERNAL ASSESSMENT-1st , 2019-20
Sub: MATHEMATICS, Course – C2

Full Marks: 10 **Time: 30 m.**
Answer any five questions: $(2 \times 5 = 10)$

1. Determine the conditions for which the system of equations has many solutions.
$$x + y + z = b$$
$$2x + y + 3z = b + 1$$
$$5x + 2y + az = b^2$$
2. Examine the nature of intersection of the triad of planes.
$x + y - z = 3, 5x + 2y + z = 1, 2x + 2y - 2z = 1;$
3. Use Cayley-Hamilton theorem to find A^{-1}, where A=$\begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}$.
4. If λ be an eigen value of a real orthogonal matrix A. Prove that $\frac{1}{\lambda}$ is also an eigen value of A.
5. If n be an integer, prove that $\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n = \cos\left(\frac{n\pi}{2} - n\theta\right) + i\sin\left(\frac{n\pi}{2} - n\theta\right)$.
6. Prove that if n be composite then $2^n - 1$ is composite.
7. Prove that $(A \cup B)^c = A^c \cap B^c$.
8. Prove that the roots of the equation are all real –
$\frac{1}{x+a_1} + \frac{1}{x+a_2} + \cdots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots + \frac{1}{x+a_n} = \frac{1}{x}$, where $a_j, j = 1,2,3, \ldots\ldots, n$ are all real positive numbers.

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – I , INTERNAL ASSESSMENT-2nd , 2019-20
Sub: MATHEMATICS, Course – C2

Full Marks: 10 **Time: 30 m.**

Answer any five questions: $(\mathbf{2 \times 5 = 10})$

1. Examine if the set S = $\{(x, y, z) \in \mathbb{R}^3 : xy = z\}$ is a subspace of $\mathbb{R}^3$.
2. Find a basis for the vector space$\mathbb{R}^3$, that contains the vectors (1,0,1) and (1,1,1).
3. Find the dimension of the subspace S of $\mathbb{R}^3$ defined by S = $\{(x, y, z) \in \mathbb{R}^3 : 2x + y - z = 0\}$.
4. Let $f: A \to B$ be a mapping. A relation ρ on A is defined as $x\ \rho\ y\ iff\ \ f(x) = f(y)$. Prove that ρ is an equivalence relation.
5. Let p & q are distinct primes, $a \in \mathbb{Z}$. Prove that $a^{pq} - a^p - a^q + a$ is divisible by pq.
6. Find the remainder when $1^3 + 2^3 + \cdots + 99^3$ is divided by 3.
7. If a, b, c, d be positive real numbers, each less than 1, prove that $8(abcd + 1) > (a + 1)(b + 1)(c + 1)(d + 1)$.
8. If $\alpha, \beta, \gamma, \delta$ are the roots of the polynomial equation $x^4 + px^3 + qx^2 + rx + s = 0$, prove that $\sum \alpha^2\beta = 3r - pq$.