DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – II , INTERNAL ASSESSMENT-1st , 2017-18
Sub: MATHEMATICS, Course - C3

Full Marks: 10 **Time: 30 m.**

Answer any five questions: $(\mathbf{2} \times \mathbf{5} = \mathbf{10})$

1. Chose the correct alternative with proper justification
   The set $\{x: sinx = 0\}$ has
   i) finitely many limit points ii) infinitely many limit points iii) no limit points
2. Prove that union of two open sets is an open set.
3. Let $G \subset R$ be an open set and $F \subset R$ be a closed set. Prove that $G - F$ is an open set.
4. Prove that $\lim_{n\to\infty} \sqrt[n]{n} = 1$.
5. Prove that $\lim_{n\to\infty} \left(\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots . + \frac{1}{\sqrt{n^2+n}}\right) = 1$
6. Test the Convergence of the Series $\sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{1}{n}$.
7. Show that the Series $\sum_{n=1}^{\infty} \frac{n!\, x^n}{n^n}$ $(x > 0)$ is convergent if $x < e$.

DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc(H) Sem – II , INTERNAL ASSESSMENT-2nd , 2017-18

Sub: MATHEMATICS, Course - C3

Full Marks: 10 **Time: 30 m.**

Answer any five questions: $(\mathbf{2 \times 5 = 10})$

1. Prove that every finite subset of $\mathbb{R}$ is a closed set.
2. Find out the correct alternative –"Every infinite bounded set of real numbers has a limit point".
   (a) True (b) False (c) Not always true.
3. Prove that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{n^2+1}$ is Conditionally Convergent.
4. Prove that $\lim_{n\to\infty} \frac{1+\frac{1}{3}+\frac{1}{5}+\cdots+\frac{1}{2n+1}}{2n+1} = 0.$
5. Prove that a Convergent Sequence is a Cauchy Sequence.
6. Let $\sum x_n$ be a series of Positive real numbers and $y_n = \frac{x_1+x_2+\cdots+x_n}{n}$.Then Show that $\sum y_n$ is divergent.
7. Find the value of $\sum_{n=1}^{\infty} \tan^{-1} \frac{1}{1+n^2+n}$.

DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc(H) Sem – II , INTERNAL ASSESSMENT-1st , 2018-19

Sub: MATHEMATICS, Course – C3

Full Marks: 10 **Time: 30 m.**

Answer any five questions: $(\mathbf{2 \times 5 = 10})$

1. Let G be an open subset in R and S be a subset of R such that $G \cap S = \emptyset$. Prove that $G \cap S' = \emptyset$.
2. Prove that the intersection of a finite number of closed sets in R is a closed set.
3. Define closure of a set in R. Let A be a non – empty subset of R and $d(x, A) = inf\{|x - y|: y \in A\}$. Prove that $d(x, A) = 0$ if and only if $x \in \bar{A}$. Here x is real.
4. Prove that if $x_n = (a^n + b^n)^{\frac{1}{n}}$ & $0 < b < a$, show that $\lim_{n\to\infty} x_n = a$
5. Prove that the sequence $\{u_n\}$ where $u_n = \frac{n!}{n^n}$ is a null sequence.
6. Let $\{u_n\}$ be a bounded sequence and $\lim_{n\to\infty} v_n = 0$. Prove that $\lim_{n\to\infty} u_n v_n = 0$.
7. Using Cauchy's criterion prove that the series $\mathbf{1}+\frac{\mathbf{1}}{\mathbf{2}}+\frac{\mathbf{1}}{\mathbf{3}}+\cdots \ldots \ldots \ldots \ldots$ diverges.
8. Test the convergence of the series $\frac{\mathbf{1+2}}{\mathbf{2^3}}+\frac{\mathbf{1+2+3}}{\mathbf{3^3}}+\frac{\mathbf{1+2+3+4}}{\mathbf{4^3}}+\cdots \ldots \ldots \ldots \ldots$

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – II , INTERNAL ASSESSMENT-2nd , 2018-19
Sub: MATHEMATICS, Course – C3

Full Marks: 10 **Time: 30 m.**

Answer any five questions: $(\mathbf{2 \times 5 = 10})$

1. Let A and B be subsets of R of which A is closed and B is compact. Prove that $A \cap B$ is also compact subset of R.
   [**Compact Set in *R***: A set is said to be a compact if every open cover of the set has a finite sub-cover]
2. Prove that the set of all circles in the plane having rational radii and centres with rational coordinates is an enumerable set.
3. If a boundary point of a set S is not a point of S prove that it is limit point of the set.
4. Show that for any fixed value of x, the series $\sum_{n=1}^{\infty} \frac{\sin(nx)}{n^2}$ is convergent.
5. When is a series said to converge conditionally?
6. Prove that a bounded sequence $\{u_n\}$ is convergent iff $\overline{lim} u_n = \underline{lim} u_n$.
7. Prove that $\lim_{n\to\infty} \frac{(n!)^{\frac{1}{n}}}{n} = \frac{1}{e}$.
8. If $\{u_n\}$ be a Cauchy sequence in $\mathbb{R}$ having a subsequence converging to a real number l, Prove that $\lim_{n\to\infty} u_n = l$.