DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – III , INTERNAL ASSESSMENT-1st , 2018-19
Sub: MATHEMATICS, Course – C5

Full Marks: 10 **Time: 30 m.**
Answer any five questions: $(\mathbf{2 \times 5 = 10})$

1. Prove that $\lim_{x \to 0} x \sin \frac{1}{x^2} = 0$.
2. State and Prove the "Sandwich Theorem" for the limit of the real valued functions and of real variables.
3. Use Taylor's theorem to Prove that $\cos x \geq 1 - \frac{x^2}{2}$ for $-\pi < x < \pi$.
4. Justify that the function defined by $f(x) = x^2 \sin \frac{1}{x}$, $x \neq 0$
   = 0 , $x = 0$ is differentiable at x =0.
5. Expand $f(x) = a^x$ in a finite series with Lagrange's form of remainder.
6. Show that in Cauchy's Mean Value Theorem if $f(x) = e^x$ & $g(x) = e^{-x}$ then θ is independent of both x and h. Find the value of θ.
7. Prove that if $f: \mathbb{R} \to \mathbb{R}$ is an even function and has a derivative at every point, then the derivative $f^{/}$ is an odd function.
8. The function defined by $f(0) = 0$ & $f(x) = 0$, $x \in \mathbb{R} \backslash \mathbb{Q}$
   $= \frac{1}{q}$, $x = \frac{p}{q}$, $q \in \mathbb{N}, p \in \mathbb{Z}$ & $\gcd(p, q) = 1$

   Show that f is not differentiable at $x = 0$.

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc.(H) Sem. – III , INTERNAL ASSESSMENT-2nd , 2018-19
Sub: MATHEMATICS, Course – C5

Full Marks: 10 **Time: 30 m.**
Answer any five questions: **($2 \times 5 = 10$)**

1. Use Taylor's theorem to prove that $\cos x \geq 1 - x^2$ for $-\pi < x < \pi$.
2. Show that continuity of a function f at a point c is necessary but not sufficient condition for the existence of the derivative of f at c.
3. A function $f: \mathbb{R} \Rightarrow \mathbb{R}$ satisfies the condition $|f(x) - f(y)| \leq (x - y)^2 \ \forall \ x, y \in \mathbb{R}$. Prove that f is a constant function on $\mathbb{R}$.
4. Prove that the function $(x-1)^3 + (x-2)^3 + (x-3)^3 +$ ..... $+(x-2018)^3 = 0$ has only one real root.
5. Let a function $f: [a, \infty) \to \mathbb{R}$ be twice differentiable on $[a, \infty)$ and $\exists \ A, B > 0$ such that $|f(x)| \leq A$, $|f^{//}(x)| \leq B \ \ \forall \ x \in [a, \infty)$. Prove that $|f^{/}(x)| \leq 2\sqrt{AB} \ \ \forall \ x \in [a, \infty)$.
6. Write down the Maclaurin's theorem related to expansion of functions with Cauchy's form of remainder.
7. Let A and B are subsets of R and $f: A \to R$ be continuous on A and let $g: B \to R$ be continuous on B such that $f(A) \subset B$. Prove that the composite function $g_o f: A \to R$ is a continuous function on A.
   State your comment over the converse of the above result.
8. A function f is defined on R by $f(x) = \begin{cases} \cos\frac{1}{x^2}, x \neq 0 \\ 0, x = 0 \end{cases}$. Prove that the function is not continuous at 0.

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – III , INTERNAL ASSESSMENT-1st , 2019-20
Sub: MATHEMATICS, Course – C5

Full Marks: 10 **Time: 30 m.**

Answer any five questions: $(\mathbf{2 \times 5 = 10})$

1. Prove that in a metric space (X,d) $|d(x,y)-d(a,b)| \leq d(a,x)+d(b,y)\ \forall\ x,y,a,b \in X$.
2. Prove that (l_p, d) with $p \geq 1$ is a metric space where $d(x,y) = \left(\sum_{n=1}^{\infty} |x_n - y_n|^p\right)^{\frac{1}{p}}$ where $x = \{x_n\}, y = \{y_n\}$.
3. Prove that every open subset of a discrete metric space is open.
4. Draw $B_d((0,0),1)$ where $d(x,y) = \max\{|x_1 - y_1|, |x_2 - y_2|\}$ where $x = (x_1, x_2), y = (y_1, y_2) \in \mathbb{R}^2$.
5. Let $f: (-1,1) \to \mathbb{R}$ be continuous at 0.If $f(x) = f(x^2)\ \forall\ x \in (-1,1)$. Prove that $f(x) = 0\ \forall\ x \in (-1,1)$.
6. Let $f: \mathbb{R} \to \mathbb{R}$ be continuous on $\mathbb{R}$. Prove that for every open subset G of $\mathbb{R}$ $f^{-1}(G)$ is open in $\mathbb{R}$.
7. Evaluate (i) $\lim_{x \to 0} [\frac{\sin x}{x}]$ (ii) $\lim_{x \to 0} \frac{1}{1+e^{\frac{1}{x}}}$.
8. Prove that the function $f(x) = \frac{1}{x}$, $x \in (0,1)$ is not uniformly continuous on $(0,1)$.

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc.(H) Sem. – III , INTERNAL ASSESSMENT-2nd , 2019-20
Sub: MATHEMATICS, Course – C5

Full Marks: 10 **Time: 30 m.**

Answer any five questions: **$(2 \times 5 = 10)$**

1. Show that the equation $x \ln x = 3 - x$ has at least one root in $(1,3)$.
2. Is there a function F such that $F^{/}(x) = f(x)$ in $[-1,1]$ where $f(x) = \begin{cases} 0, -1 \le x \le 0 \\ 1, 0 < x \le 1. \end{cases}$
3. Using Maclaurin's series to show that $\sin x > x - \frac{x^3}{6}, 0 < x < \frac{\pi}{2}$.
4. Using LMVT to prove that $\frac{b-a}{\sqrt{1-a^2}} < \sin^{-1} b - \sin^{-1} a < \frac{b-a}{\sqrt{1-b^2}}, 0 < a < b < 1$.
5. Prove that f is discontinuous at all irrational points where $f(x) = \begin{cases} x, & x \in \mathbb{Q} \\ 1-x, & x \in \mathbb{R} \setminus \mathbb{Q} \end{cases}$
6. Using $\partial A = \bar{A} \cap \overline{(X \setminus A)}$ Prove that $int\ A \cup ext\ A \cup \partial A = X$.
7. Prove that $int\ A$ is the largest open set contained in A. Hence prove that interior operator from $P(X) to\ P(X)$ is an idempotent one.
8. Prove that $\bar{A} = \{x \in X : d(x, A) = 0\}$ & $int(X \setminus A) = \{x \in X : d(x, A) > 0\}$.