DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – III , INTERNAL ASSESSMENT-1st , 2018-19
Sub: MATHEMATICS, Course – C6

Full Marks: 10 **Time: 30 m.**
Answer any five questions: $(\mathbf{2 \times 5 = 10})$

1. Let G be a commutative group. Prove that the set H = $\{a \in G : o(a)\ divides\ 10\}$ is a subgroup of G.
2. Find the elements of order 5 in Z_{20}.
3. Let $S = \{1, \alpha, \alpha^2, \ldots, \alpha^5\}$ be a group under multiplication. Show that $(S,\ .)$ be a cyclic group where $\alpha = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$.
4. Prove that a cyclic group of prime order has no proper non-trivial subgroup.
5. Verify that whether the pair of groups $(\mathbb{R}^+, .)$ and $(\mathbb{R}, +)$ are isomorphic or not.
6. Prove that if a group G has a unique subgroup of order 2018 then that subgroup is a Normal subgroup of G.
7. Prove that order of the group $Aut(\mathbb{Z}_p)$ is $p - 1$, p is a prime.
8. Find the Centre of Dihedral group of Order 8.

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – III , INTERNAL ASSESSMENT-2nd , 2018-19
Sub: MATHEMATICS, Course – C6

Full Marks: 10 **Time: 30 m.**

Answer any five questions: $(\mathbf{2 \times 5 = 10})$

1. Let G be a group and A be a non-empty subset of G. Define the normalizer of A in G and show that the normalizer of A in G is a subgroup of G.
2. Find the order of the permutation $(1\ 2\ 3)(5\ 6)\ in\ S_6$.
3. Let G be an infinite cyclic group generated by a. Prove that $a\ and\ a^{-1}$ are the only generators of the cyclic group.
4. Show that $(\mathbb{Z}_4, +)$ is a cyclic group. Also find its generators.
5. Let $G\ and\ G^{/}$ be two finite groups and $\theta: G \to G^{/}$ be an epimorphism. Prove that $o\left(G^{/}\right)|o(G)$.
6. Prove that the additive groups $\mathbb{Z}\ and\ \mathbb{Z} \times \mathbb{Z}$ are not isomorphic.
7. Find the number of elements of order 6 in the group $\mathbb{Z}_6 \times \mathbb{Z}_4$.
8. If $\gcd(m, n) = 1$, Prove that $\mathbb{Z}_m \times \mathbb{Z}_n \cong \mathbb{Z}_{mn}$ ..

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – III , INTERNAL ASSESSMENT-1st , 2019-20
Sub: MATHEMATICS, Course – C6

Full Marks: 10 **Time: 30 m.**

Answer any five questions: $(\mathbf{2 \times 5 = 10})$

1. Let G be a commutative group. Prove that the set H = $\{a \in G : o(a)\ divides\ 15\}$ is a subgroup of G.
2. Find the elements of order 5 in Z_{10}.
3. Prove that nth roots of unity form a cyclic group under multiplication.
4. Prove that a group of prime order is cyclic.
5. Let G be a group and $a \in G$ such that $o(a) = n$ & $a^m = e$ for some $m \in \mathbb{N}$. Prove that $n|m$.
6. Let G be a group and $Z(G) = \{x \in G : gx = xg\ \forall\ g \in G\}$. Prove that $Z(G)$ is a subgroup of G.
7. Prove that in a group G $a^2 = e\ \forall\ a \in G$. Prove that G is Abelian.
8. Prove that $(\mathbb{Z},*)$ is a group where $*$ is defined by $a * b = a + b + 1\ \forall\ a, b \in \mathbb{Z}$.

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – III , INTERNAL ASSESSMENT-2nd, 2019-20
Sub: MATHEMATICS, Course – C6

Full Marks: 10 **Time: 30 m.**
Answer any five questions: $(\mathbf{2 \times 5 = 10})$

1. Prove that a group of order 27 must have a subgroup of order 3.
2. Let H be a subgroup of a group G and $[G : H] = 2$ then show that H is a normal in G.
3. Prove that if a group G has a unique subgroup H of order 2019 then show that H is a Normal in G.
4. If H be a subgroup of a commutative group G then prove that the quotient group G/H is commutative.
5. Find all homomorphisms from the group $(\mathbb{Z}_6,+)$ to $(\mathbb{Z}_4, +)$.
6. Verify that whether the groups $(\mathbb{Z}_6, +)$ and S_3 are isomorphic or not.
7. Let G be a group of order 9 and H be a group of order 6. Show that there does not exist a homomorphism of G onto H.
8. Find the Centre of Dihedral group of order 8.