DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE

B.Sc (Honours) Sem - IV , 1st INTERNAL ASSESSMENT, 2018-19
Sub: MATHEMATICS, Paper- CC 8

Full Marks: 10 **Time**: 30 m.

Answer the following questions: $(\mathbf{2} \times \mathbf{5} = \mathbf{10})$

01. Let $f: [a, b] \to R$ be a bounded function on $[a, b]$ and P be a partition of $[a, b]$. Then prove the following inequality - $L(P, f) \leq U(P, f)$.
[***Symbols have their usual meaning***]

02. Let $f: [a, b] \to R$ be a bounded function on $[a, b]$ and P, Q be any two partitions of the interval $[a, b]$. Prove that $L(P, f) \leq U(Q, f)$.

03. Let us consider the function $f: [0,1] \to R$ defined as $y = f(x) = \begin{cases} 1, \text{ if } x \text{ is rational} \\ 0, \text{ if } x \text{ is irrational} \end{cases}$
Show that the function is not ***Riemann Integrable*** on $[0,1]$.

04. Let a function $f: [a, b] \to R$ be continuous on the closed and bounded interval $[a, b]$. Then prove that the function is ***Riemann Integrable*** function on $[a, b]$.

05. Let $f: [a, b] \to R$ be bounded and monotone increasing function on the closed and bounded interval $[a, b]$. If P_n be a partition of the interval $[a, b]$ dividing into n number of sub – intervals of equal length, prove that $\int_a^b f \leq U(P_n, f) \leq \int_a^b f + b-anfb-fa$.

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE

B.Sc (Honours) Sem - IV, 2^{nd} INTERNAL ASSESSMENT, 2018-19
Sub: MATHEMATICS, Paper- CC 8

Full Marks: 10 **Time**: 30 m.

Answer the following questions: $(\mathbf{2 \times 5 = 10})$

01. Let $f:[a,b] \to R$ be defined as $f(x) = \begin{cases} a_{n+1}, \text{ if } x = n \in [0,2019] \cap Z \\ 0, \text{otherwise} \end{cases}$. Prove that the function f is Riemann Integrable and evaluate $\int_0^{2019} f$.
[***Symbols have their usual meaning***]

02. Let $f(x) = [x], x \in [1,3]; \varphi(x) = \begin{cases} x, x \in [1,2] \\ 2x-2, x \in]2,3] \end{cases}$. Show that the given function f is Riemann Integrable function and without evaluating the integral show that $\int_1^3 f = \varphi(3) - \varphi(1)$.

03. Let $f,g:[a,b] \to R$ be both Riemann Integrable functions on $[a,b]$. Prove that $max(f,g):[a,b] \to R$ is also Riemann Integrable function.

04. For each $n \in N$, let $f_n(x) = x - \frac{1}{n}, g_n(x) = x + \frac{2}{n}, 0 \le x < \infty$. Show that the given sequences are uniformly convergent on $[0,\infty[$. Determine the nature of the sequence $\{f_n g_n\}$.

05. Prove that the uniform limit of a sequence of continuous functions is continuous on the same domain of definition.

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[***Symbols have their usual meaning***]

02. Let $f(x) = [x], x \in [1,3]; \varphi(x) = \begin{cases} x, x \in [1,2] \\ 2x-2, x \in]2,3] \end{cases}$. Show that the given function f is Riemann Integrable function and without evaluating the integral show that $\int_1^3 f = \varphi(3) - \varphi(1)$.

03. Let $f,g:[a,b] \to R$ be both Riemann Integrable functions on $[a,b]$. Prove that $max(f,g):[a,b] \to R$ is also Riemann Integrable function.

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