DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – IV , INTERNAL ASSESSMENT-1st , 2018-19
Sub: MATHEMATICS, Course – C9

Full Marks: 10 **Time: 30 m.**

Answer any five questions: $(\mathbf{2 \times 5 = 10})$

1. Let $f(x,y) = \sqrt{|xy|}$ .Prove that f is not differentiable at $(0,0)$.
2. Verify whether $\lim_{(x,y)\to(0,0)} \frac{|x|}{y^2} e^{-\frac{|x|}{y^2}}$ exists or not.
3. If $V = \ln(x^3 + y^3 + z^3 - 3xyz)$, Prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right) V = \frac{3}{x+y+z}$.
4. Show that the function f is continuous at $(0,0)$ where $f(x,y) = \begin{cases} \frac{x^3 - y^3}{x^2+y^2}, x^2 + y^2 \neq 0 \\ 0, x^2 + y^2 = 0 \end{cases}$
5. Let z be a differentiable function of x & y and let $x = r\cos\theta$, $y = r\sin\theta$.
   Prove that $\left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2 = \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$.
6. Find the work done in moving a particle once around a circle C in the xy plane, if the circle has the centre at $(0,0)$ and radius 2 unit and the field is given by
   $\vec{F} = (2x - y + 2z)\hat{\imath} + (x + y - z^2)\hat{\jmath} + (3x - 2y - 5z)\hat{k}$.
7. Show that $\vec{F} = (2xy + z^3)\hat{\imath} + (x^2)\hat{\jmath} + (3xz^2)\hat{k}$ is a conservative force field.
8. Find the maximum value of the directional derivative of the scalar point function
   $\emptyset(x,y,z) = x^2 - y^2 + z^2$ at $(1,3,2)$. Find also the direction in which it occurs.

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – IV , INTERNAL ASSESSMENT-2nd , 2018-19
Sub: MATHEMATICS, Course – C9

Full Marks: 10 **Time: 30 m.**

Answer any five questions: $(\mathbf{2} \times \mathbf{5} = \mathbf{10})$

1. Compute the Surface area of the unit Sphere.
2. Assuming that the inversion of order of integration is possible, change the order of integration $\int_{\frac{1}{3}}^{\frac{2}{3}} dx \int_{x^2}^{\sqrt{x}} f(x,y)dy.$
3. If $u = x + y + z, uv = y + z, z = uvw, Show\ that\ \frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v.$
4. If $F(x,y,z) = 0$, Prove that $(\frac{\partial x}{\partial y})_z (\frac{\partial y}{\partial z})_x (\frac{\partial z}{\partial x})_y = -1.$
5. Find all the stationary points of the function $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x.$
6. Prove that $ax^2 + 2hxy + by^2$ & $Ax^2 + 2Hxy + By^2$ are independent unless $\frac{a}{A} = \frac{h}{H} = \frac{b}{B}$.
7. Show that the area bounded by the simple closed curve C is given by $\frac{1}{2}\oint xdy - ydx.$
8. Show that $\iint \vec{r}.\vec{n} ds = 3V$ where V is the volume of the closed Surface S.