DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE
B.Sc(H) Sem – V , INTERNAL ASSESSMENT-1st , 2019-20
Sub: MATHEMATICS, Course – C11

Full Marks: 10 **Time: 30 m.**

Answer any five questions: $(\mathbf{2} \times \mathbf{5} = \mathbf{10})$

1. If ω be the angular velocity of a planet at the nearer end of the major axis, prove that its period is $\frac{2\pi}{\omega}\sqrt{\frac{(1+e)}{(1-e)^3}}$.
2. Write down Kepler's 2nd law on planetary motion & deduce the expression for the periodic time of a planet.
3. Prove that $h = pv$ where h, p, v are the standard notations.
4. A particle describes the parabola $p^2 = ar$ under a force which is always directed towards its focus. Find the law of force.
5. Form PDE by eliminating the function from $z = e^{ax+by} f(ax - by)$.
6. Find the integral surface of the linear PDE $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0, z = 1$.
7. Find the complete integral of $zpq = p + q$.
8. Prove that along every characteristic strip of the PDE $f(x, y, z, p, q) = 0$ the function f is constant.

DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc(H) Sem – V , INTERNAL ASSESSMENT-2nd , 2019-20

Sub: MATHEMATICS, Course – C11

Full Marks: 10 **Time: 30 m.**

Answer any five questions: $(\mathbf{2\times 5 = 10})$

1. Classify $u_{xx} + u_{yy} + u_{zz} = 0$
2. State Cauchy- Kowalewskaya Theorem.
3. Consider the following Cauchy problem of an infinite string with initial condition
$$u_{tt} - c^2 u_{xx} = 0, x \in \mathbb{R}, t > 0$$
$$u(x,0) = f(x), x \in \mathbb{R} \ \& \ u_t(x,0) = g(x), x \in \mathbb{R}.$$
Write down the corresponding characteristic equation and the integrals.
4. Consider the Cauchy problem for $u_{tt} = c^2 u_{xx} + h^*(x,t)$ with initial conditions$u(x,0) = f(x),\ u_t(x,0) = g^*(x)$. Show that by the coordinate transformation $y = ct$ the above problem reduced to $u_{xx} - u_{yy} = h(x,y); u(x,0) = f(x), u_y(x,0) = g(x)\ where\ h(x,y) = -\frac{h^*(x,t)}{c^2}\ , g(x) = \frac{g^*(x,t)}{c}$.
5. A spherical drop of liquid falling freely in a vapour acquires mass by condensation at a constant rate k. Show that the velocity after falling from rest in time t is $\frac{1}{2}gt\left(1 + \frac{M}{M+kt}\right)$.
6. A smooth parabolic tube is placed vertex downwards, in a vertical plane. A particle slides down the tube from rest under gravity. Write down the equation of motion along the tangent.
7. A point moves along the arc of a cycloid in such a manner that the tangent as it rotates with constant angular velocity. Show that the acceleration of the moving point is constant in magnitude
8. A heavy particle slides down a rough cycloid of which the coefficient of friction is μ. Its base is horizontal & vertex downwards. Write down the equations of motion.