# DEPT. OF MATHEMATICS <br> JHARGRAM RAJ COLLEGE <br> B.Sc(H) Sem - III , INTERNAL ASSESSMENT-1 ${ }^{\text {st }}$, 2019-20 <br> Sub: MATHEMATICS, Course - C12 

Full Marks: 10
Time: $\mathbf{3 0} \mathbf{~ m}$.
Answer any five questions:

1. Prove that the mapping $f: U(16) \rightarrow U(16)$ defined by $f(x)=x^{3}$ is an Automorphism.
2. Prove that a group $G$ is Abelian iff $G^{/}=\left\{e_{G}\right\}$.
3. Prove that $\mathrm{Z}(\mathrm{G})$ is a characteristic subgroup of G .
4. Find the order of $\operatorname{Inn}(\mathrm{G})$ where $\mathrm{G}=S_{3}$.
5. In $\mathbb{Z}_{30} \times \mathbb{Z}_{60}$ find two subgroups of order 12 .
6. Find the number of non-isomorphic Abelian group of order 360 .
7. Find the order of $(10,15,24)$ in $\mathbb{Z}_{12} \times \mathbb{Z}_{30} \times \mathbb{Z}_{40}$.
8. Find all Abelian groups of order $p^{3} q^{2}$, where $p, q$ are distinct primes.
9. Let H be a subgroup of order 11 and index 4 of a group G . Show that H is a normal subgroup of G .
10. Find the class equation for $s_{3}$.
11. Let G be a finite group that has only two conjugate classes. Show that $|G|=2$.
12. Show that $A_{4}$ has no subgroup of order 4 .
13. Let G be a noncommutative group of order $p^{3}, p$ a prime. Prove that $|Z(G)|=P$.
14. How many elements of order 7 are there in a group of order 28 ?
15. Show that every commutative group of order 36 contains an element of order 6 .
16. Prove Cayley's theorem by using extended cayley's theorem.
