DEPT. OF MATHEMATICS

JHARGRAM RAJ COLLEGE

B.Sc. (Honours) Sem. - V, 1st INTERNAL ASSESSMENT, 2019-20

Sub: MATHEMATICS, Paper - DSE2

Full Marks: 10 **Time**: 30 m.

Answer any five of the following questions: $(5\times \mathbf{2} = \mathbf{10})$

01. Prove the following identity –
$\lim_{x\to a+0} F(x) = F(a)$, where $F(x)$ is the distribution function of a random variable X connected to a random experiment R.E.

02. Let X be continuous random variable with probability density function $f(x)$ is given by –
$f(x) = \begin{cases} kx, 0 \le x \le 1 \\ k, 1 \le x \le 2 \\ -kx + 3k, 2 \le x \le 3 \\ 0, x > 3 \end{cases}$. Determine k and also find the distribution function $F(x)$ of the random variable.

03. Five balls are drawn from an urn containing 3 white and 7 black balls. Find the probability distribution of the number of white balls drawn without replacement.

04. If X has a gamma distribution with parameter l, find the distribution of the random variable $Y = \frac{1}{2}X^2$.

05. X is a continuous random variable with probability density function $f(x)$ given by -
$f(x) = \begin{cases} \frac{2}{x^2} \text{ if } x \ge 1 \\ 0, \text{ if } x < 1 \end{cases}$. Show that $E(X)$ exists but $E(X^2)$ does not exists.

06. Prove that the standard deviation is dependent on the unit of measurement but independent of the choice of origin of measurement.

07. If the probability density function of a random variable X is given by $f(x) = Ke^{-(x^2+2x+3)}, -\infty < x < \infty$, find the value of K and also the expectation of the random variable.

08. Find the expected value of the product of the number on n dice tossed together.

****

DEPT. OF MATHEMATICS
JHARGRAM RAJ COLLEGE

B.Sc. (Honours) Sem. - V, 2nd INTERNAL ASSESSMENT, 2019-20
Sub: MATHEMATICS, Paper - DSE2

Full Marks: 10 **Time**: 30 m.

Answer any five of the following questions: ($5\times \mathbf{2} = \mathbf{10}$)

01. If A and B are two mutually exclusive events connected to a random experiment say E and $P(A \cup B) \neq 0$, then prove that $P\left({A}/{(A \cup B)}\right) = \frac{P(A)}{P(A)+P(B)}$.

02. Prove that the distribution function of a random variable connected to a random variable say X is left discontinuous function. A coin is tossed. Write down its probability mass distribution and also calculate its distribution function. Verify the above result over the derived distribution function.

03. A bag contains 5 balls and it is not known how many of them are white. Two balls are drawn and are found to be white. What is the probability that all are white?

04. The random variable X has the following distribution function -

$$P(X = k) = 2^{-k}, k = 1,2,3, \ldots.$$

Show that, $E(X) = Var(X) = 2$.

05. If X is uniformly distributed over $[1,2]$, find U so that $P\big(X > U + E(X)\big) = \frac{1}{6}$.

06. The distribution of a random variable X is given by $P(X = -1) = \frac{1}{8}, P(X = 0) = \frac{3}{4}, P(X = 1) = \frac{1}{8}$. Verify Tchebycheff's inequality for the above mentioned distribution.

07. Let T_1 and T_2 be two unbiased estimators of the parameter θ. Under what condition $aT_1 + bT_2$ will be an unbiased estimator of the said parameter θ?

08. Prove that the maximum likelihood estimate of the parameter α of a population having density function $\frac{2}{\alpha^2}(\alpha - x), 0 < x < \alpha$ for a sample of unit size is $2x$, x being the sample value. Show also that the estimate is biased.

****