JHARGRAM RAJ COLLEGE

PHYSICS HONOURS
Assignment-1
(4th semester, 2022-23)
Paper : CC-8

1. Sketch the curves in the complex plane given by

   (a) $\text{Im}(z) = -1$

   (b) $|z - 1| = |z + i|$

   (c) $2|z| = |z - 2|$

2. Express the following in the form $x + iy$ with $x, y \in \mathbb{R}$:

   (a) $\dfrac{i}{1 - i} + \dfrac{1 - i}{i}$

   (b) all the 3rd roots of $-8i$

   (c) $\left(\dfrac{1 + i}{\sqrt{2}}\right)^{1337}$

3. Find all the complex roots of the equations:

   (a) $z^6 = -9$

   (b) $z^2 + 2z + (1 - i) = 0$

4. Write the following functions $f(z)$ in the forms $f(z) = u(x,y) + iv(x,y)$ under Cartesian coordinates with $u(x,y) = \text{Re}(f(z))$ and $v(x,y) = \text{Im}(f(z))$:

   (a) $f(z) = z^3 - z$

   (b) $f(z) = \dfrac{1}{i - z}$

   (c) $f(z) = \overline{\exp(z^2)}$

5. Suppose that $f(z) = x^2 - y^2 - 2y + i(2x - 2xy)$, where $z = x + iy$. Use the expressions

   $$x = \frac{z + \bar{z}}{2} \qquad y = \frac{z - \bar{z}}{2}$$

   to write $f(z)$ in terms of z and simplify the results.

6. Show that

   $$|\cos(z)|^2 = (\cos x)^2 + (\sinh y)^2$$

   for all $z \in \mathbb{C}$, where $x = \text{Re}(z)$ and $y = \text{Im}(z)$.

7. Find i^i and its pricipal value.

8. Show that

   $$\tanh^{-1} z = \frac{1}{2} \ln\left(\frac{1 + z}{1 - z}\right)$$

9. Compute the following limits if they exist:

   (a) $\displaystyle\lim_{z \to -i} \frac{iz^3 + 1}{z^2 + 1}$

   (b) $\displaystyle\lim_{z \to \infty} \frac{4 + z^2}{(z - 1)^2}$

   (c) $\displaystyle\lim_{z \to 0} \frac{\text{Im}(z)}{z}$

10. Show that $\displaystyle\lim_{z \to 0} \left(\frac{z}{\bar{z}}\right)$ doesn't exist.

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